Application of Robustness and Reliability Analysis

WOST 2023, 23rd of June

Marco Grosse, Bernd Büttner

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Dr.-Ing. Marco Grosse

Biography

- Lead Application Engineer
- ACE/EMEA FES, pre-sales support for Ansys optiSLang, professional services for PIDO
- more than 20 years experience with ANSYS an LS-DYNA
- working for Dynardo/Ansys since 2005
- background: civil engineering, structural mechanics and dynamics, nonlinear material behavior, PIDO

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Biography

- Manager Application Engineering for PIDO Solutions
- Working with Ansys for more than 6 years
- Background: medical engineering, optics, microfluidics (biological and chemical applications), CFD and Robust Design Optimization

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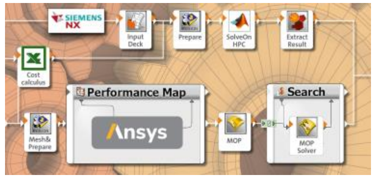
Agenda

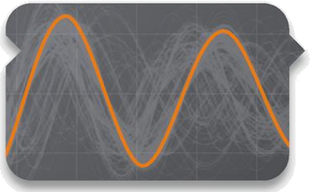
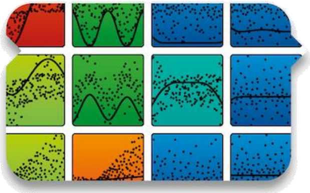
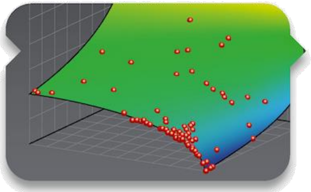
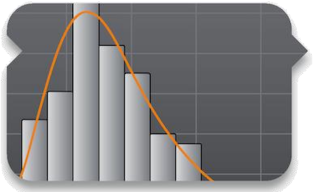
- Overview
- Simple test function for reliability methods
- ADAS/ AD application example
- Nested Robustness Evaluation in Multi-objective Design Optimization Validation
- Questions / Discussions

Ansys optiSLang for Process Integration and Design Optimization

Automation → Parametric Variation Analysis → Democratization

Automated Workflows	Model Calibration	Sensitivity Design Understanding	Optimization Design Improvement	Robustness Design Quality	Publish as web apps




Automated Workflows	Model Calibration	Sensitivity	Optimization	Robustness	Publish as web apps
Easy to build and publish repetitive workflows	Identify important model parameter for the best fit between simulation and measurement	Investigate parameter sensitivities, reduce complexity and generate best possible metamodels	Optimize design performance	Ensure design robustness and reliability	Entire organization benefits from workflows provided by CAE-experts

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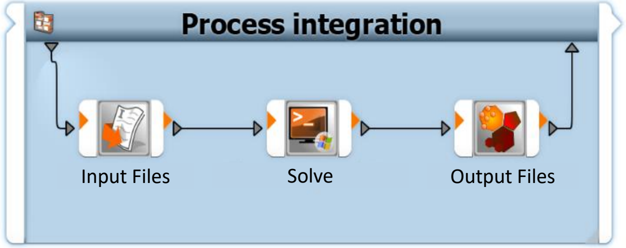
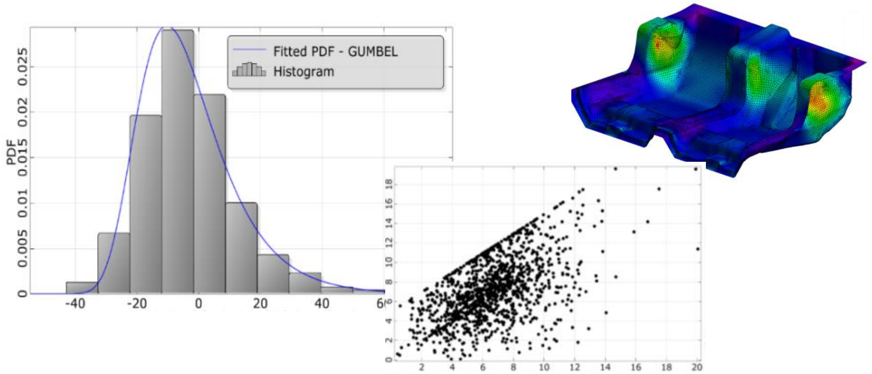
Variance and Reliability Based Robustness Analysis

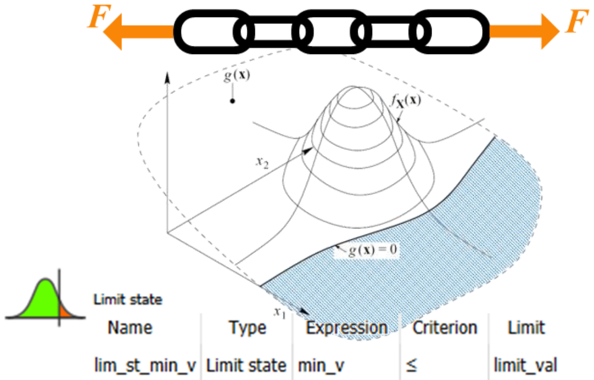
1. automate simulation workflow



2. derive and include parameter scatter and correlation

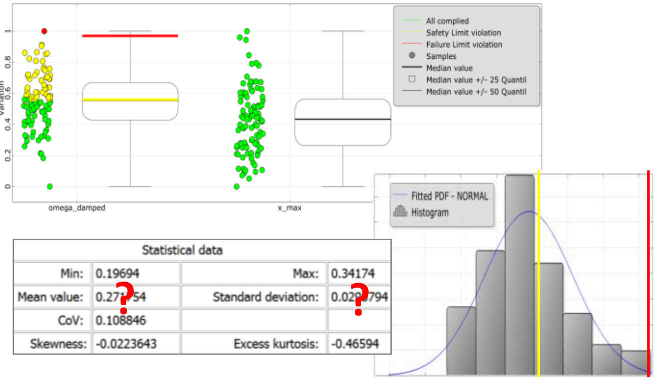


3.define limit state

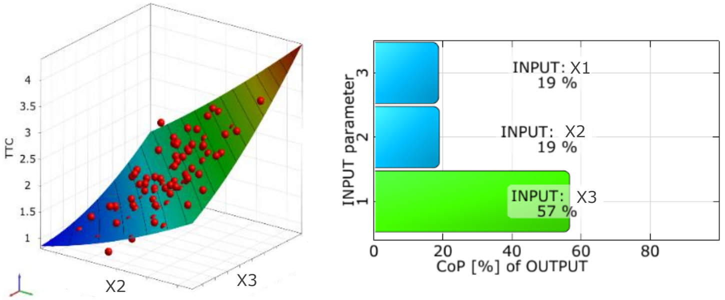


Run Analysis

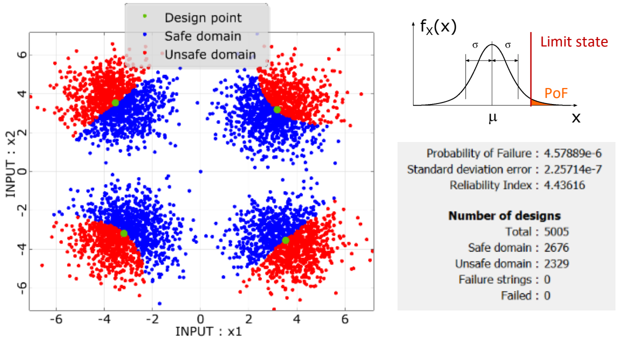
4. check the variation



5. parameter importance by robustness analysis



6. failure probability by reliability analysis



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Sensitivity and Robustness analysis for simple test function

Mishra's Bird Function

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A simple example: Mishra's Bird Function

test function used for events in advanced driver assistance systems

optimization and/or stochastic parameters

X1

X2

Xn

Mishra's Bird Function* is given as:

$$Y = f(X1, X2) =$$
$$sin(X2) \cdot e^{(1-cos(X1))^2}$$
$$+ cos(X1) \cdot e^{(1-sin(X2))^2}$$
$$+ (X1 - X2)^2$$

The domain is bounded by:

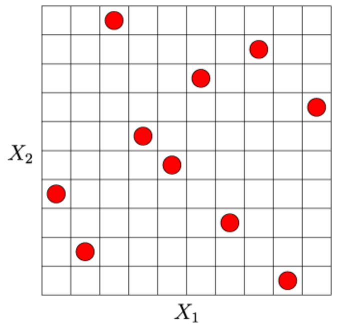
$$-10 \leq X1 \leq 0$$
$$-6.5 \leq X2 \leq 0$$

Y1

Y2

Yn

- scalar responses
- vectors
- 1D signals
- derived variables
- 2D/3D field data

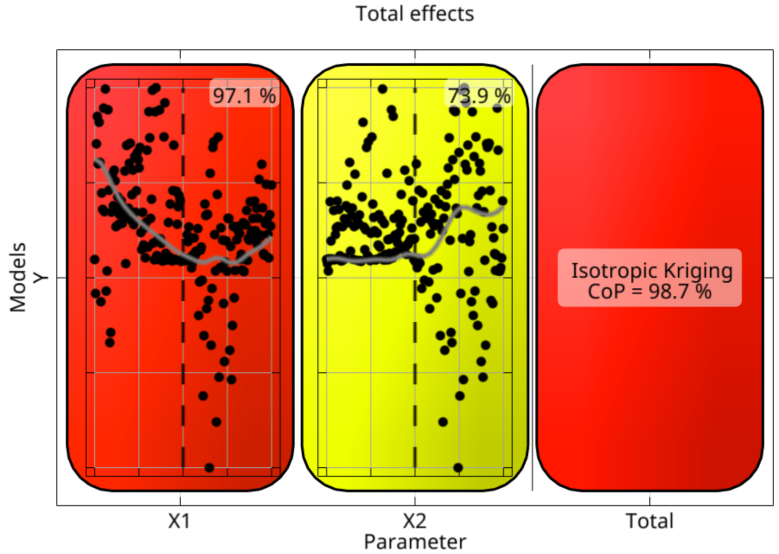


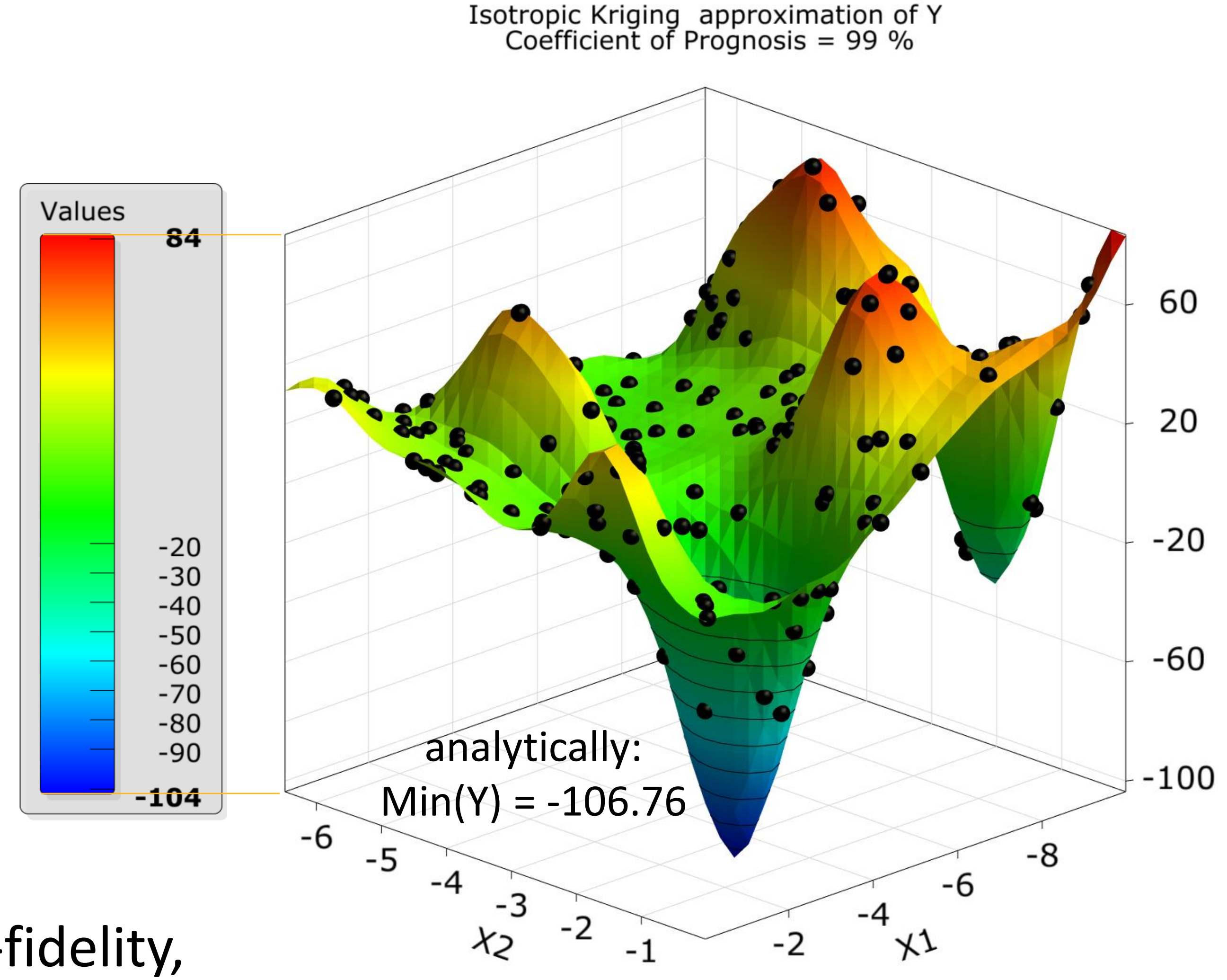
1. By DoE sampling a specific number of samples is generated and evaluated,
   Latin Hypercube Sampling: reduced sample size, decrease unwanted input correlation

[*] *Sudhanshu K Mishra. Some new test functions for global optimization and performance of repulsive particle swarm method. Available at SSRN 926132, 2006*

Sensitivity Analysis

Name	Reference value	Value type	Resolution	Range		Range plot
X1	-5	REAL	Continuous	-10	0	
X2	-3.25	REAL	Continuous	-6.5	0	



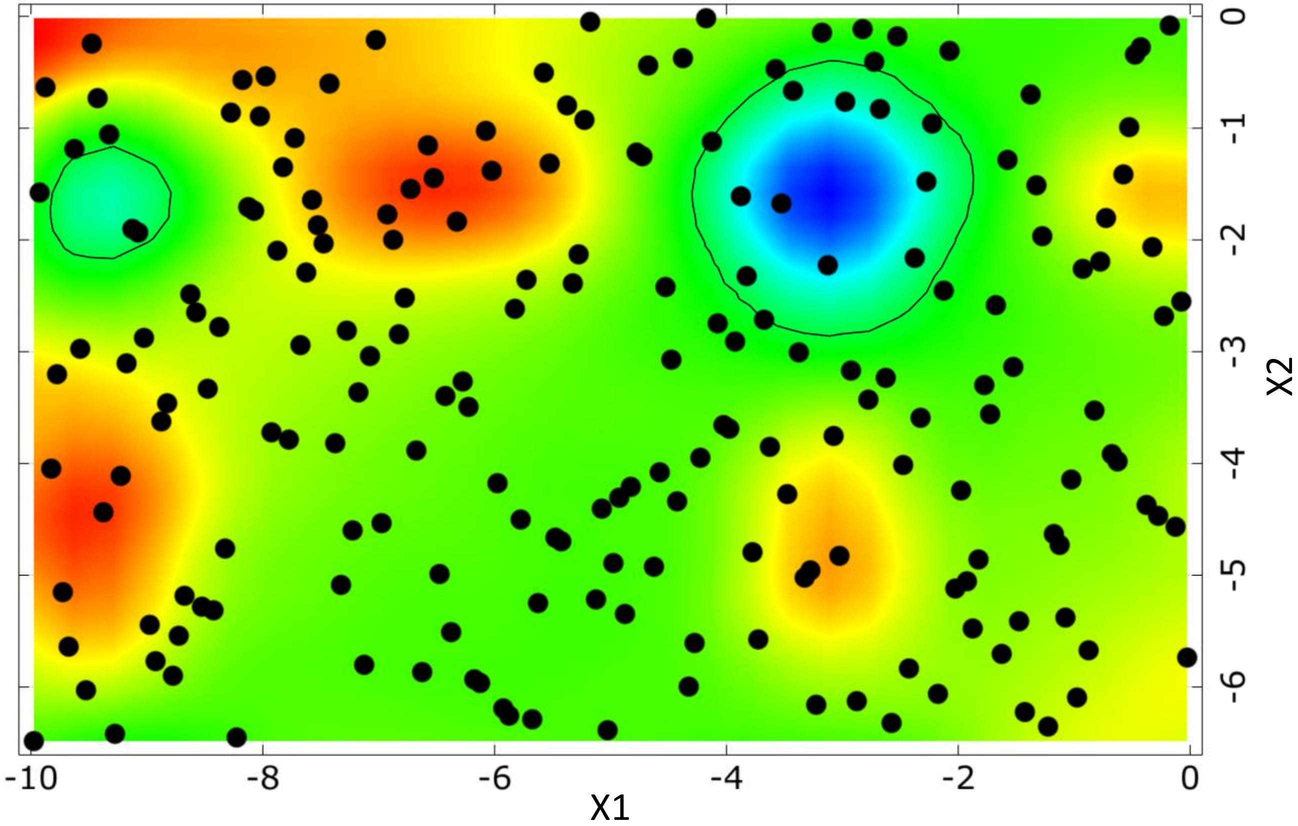


2. Responses are approximated by high-fidelity, high-precision surrogate models.
3. Parameter influence is quantified using the approximation model.

Sensitivity Analysis

Name	Reference value	Value type	Resolution	Range		Range plot
X1	-5	REAL	Continuous	-10	0	
X2	-3.25	REAL	Continuous	-6.5	0	

Name	Type	Expression	Criterion	Limit
lim_st_Y	Limit state	Y	≥	-20.0



Monte-Carlo-Sampling

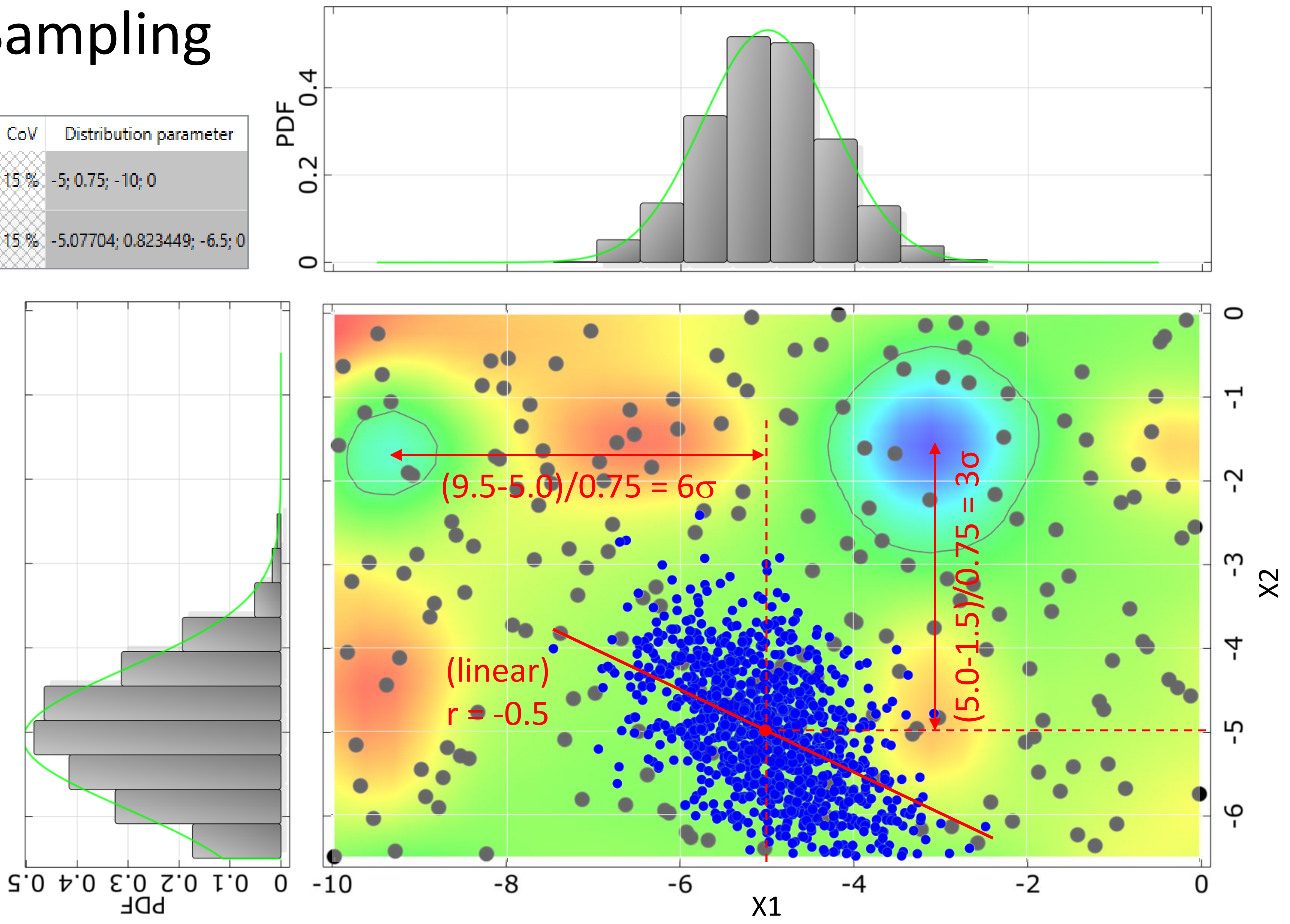
Name	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
X1		TRUNCATEDNORMAL	-5	0.75	15 %	-5; 0.75; -10; 0
X2		TRUNCATEDNORMAL	-5	0.750001	15 %	-5.07704; 0.823449; -6.5; 0

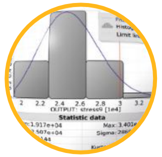
- 1000 Monte-Carlo samples

no sample above limit state

Define parameter correlations

	X1	X2
X2	0.5	1
X1	1	0.5





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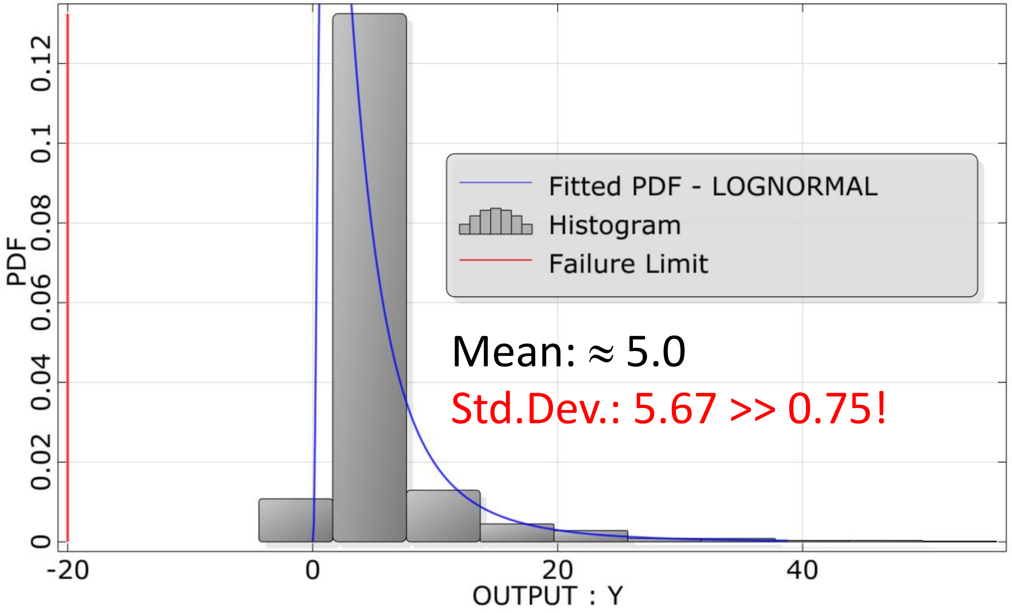
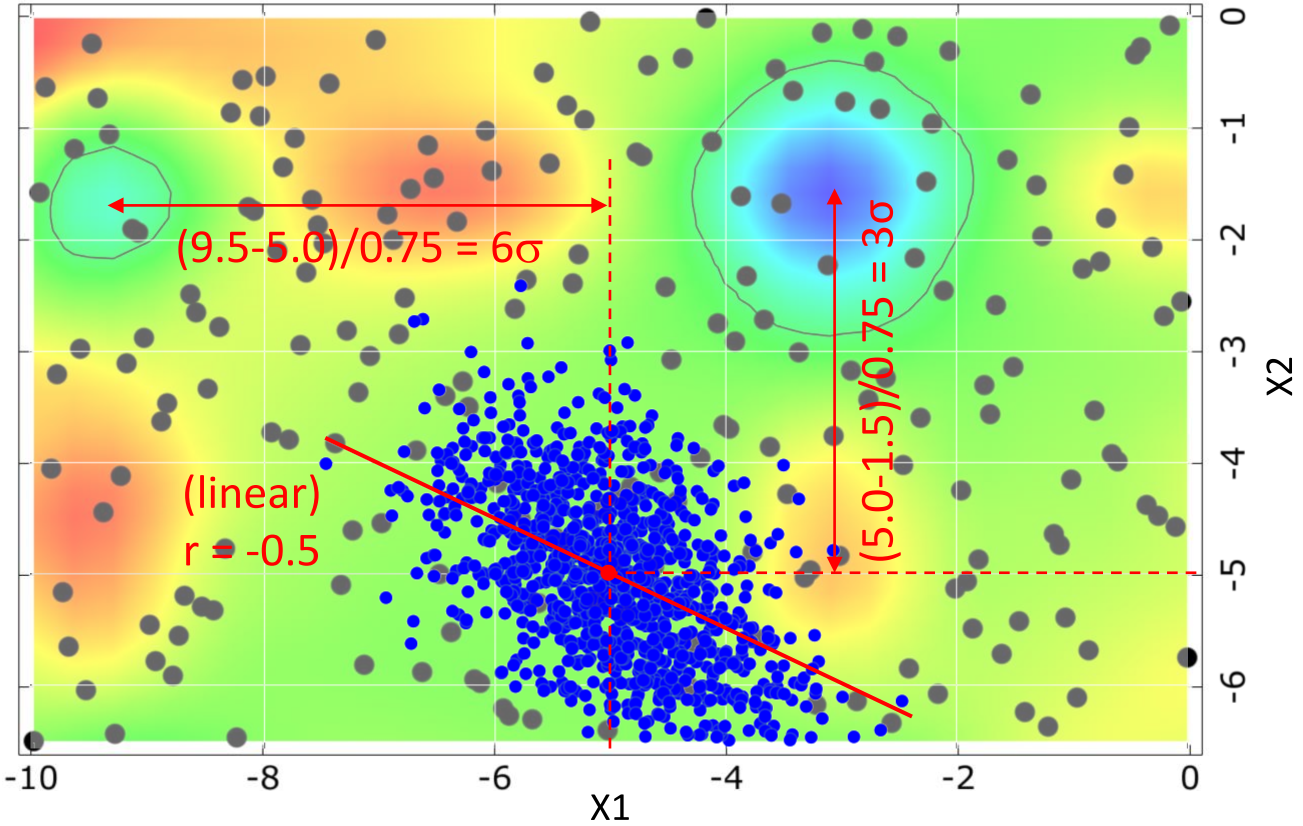
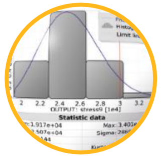
Name	Type	Expression	Criterion	Limit
lim_st_Y	Limit state	Y	≥	-20.0

- 1000 Monte-Carlo samples

=> no chance to extrapolate Probability of Failure from response probability distribution







Plain Monte-Carlo-Sampling

Name	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
X1		TRUNCATEDNORMAL	-5	0.75	15 %	-5; 0.75; -10; 0
X2		TRUNCATEDNORMAL	-5	0.750001	15 %	-5.07704; 0.823449; -6.5; 0

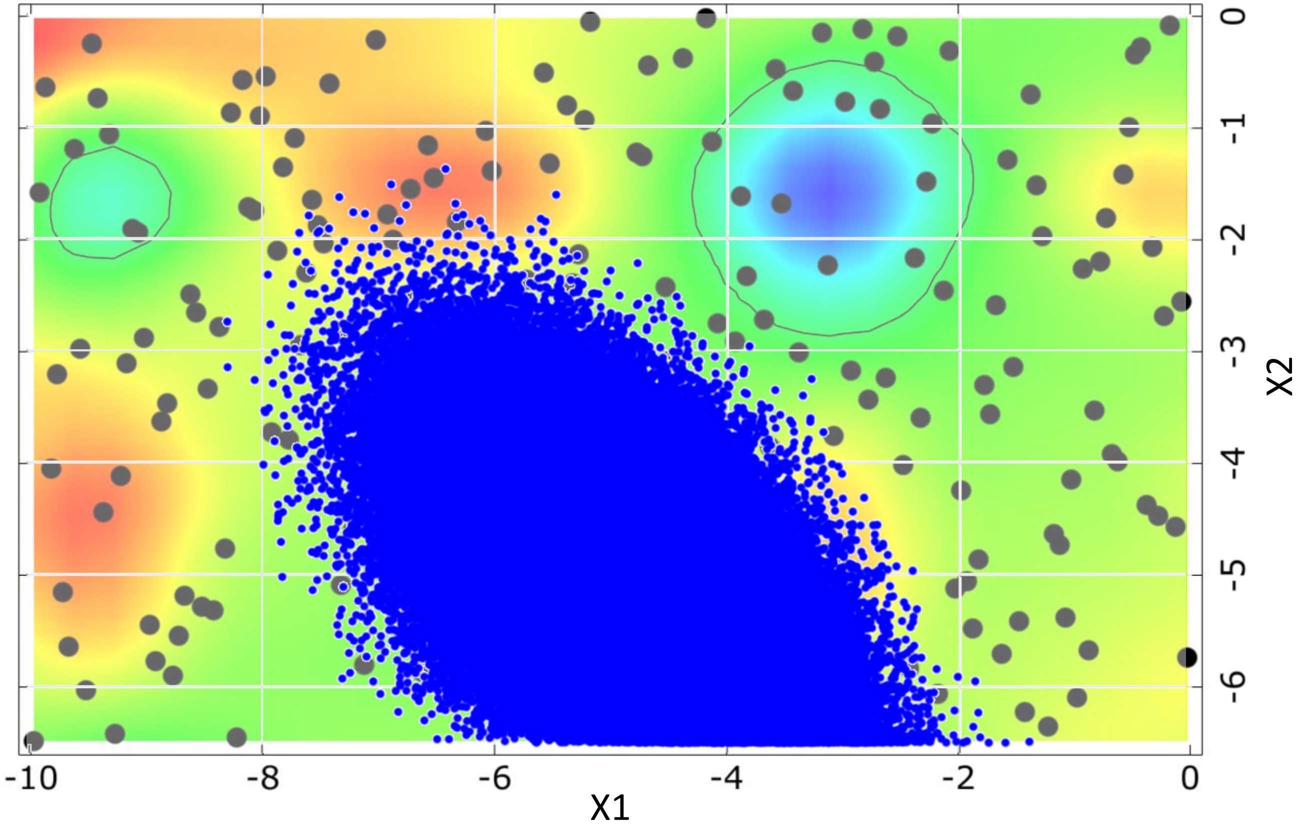
- 300’000 Monte-Carlo samples (aborted)

no sample above limit state

Monte-Carlo-Simulation,
estimation for COV=10%:

$$\mathbf{n \geq 100/PoF_{est}} \approx 100/5e\text{-}07$$

$$\mathbf{= 2.0e{+}08!}$$



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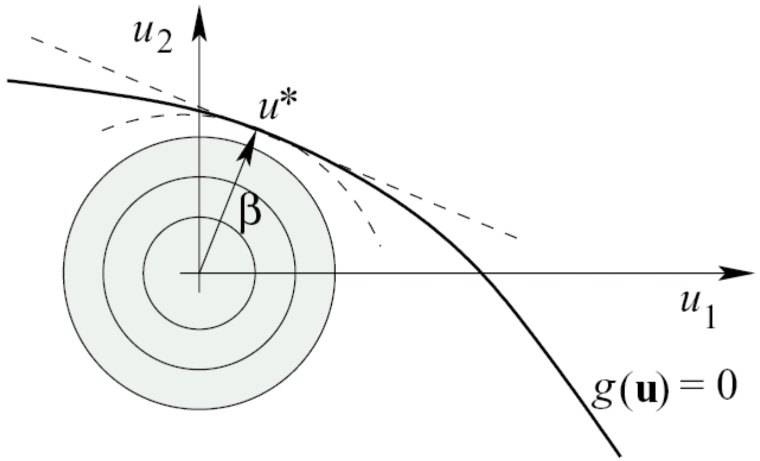
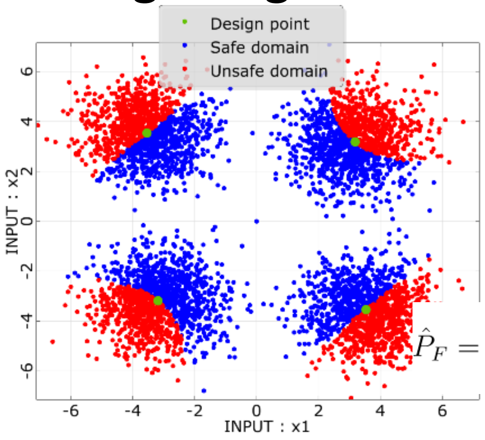
Advanced Methods for Reliability Analysis

First Order Reliability Method

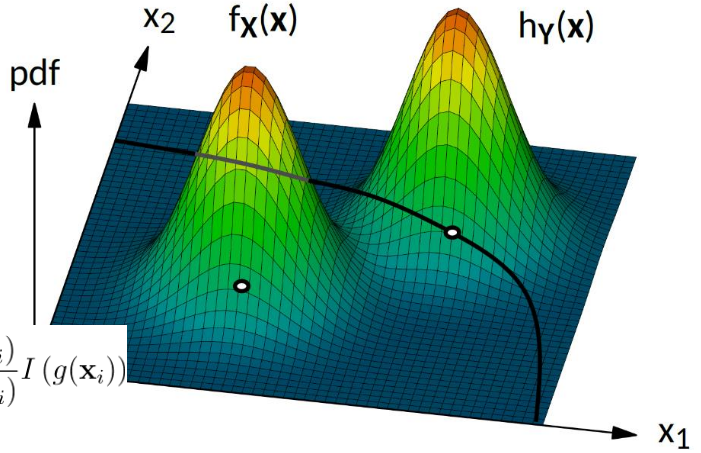


Importance Sampling using Design Point

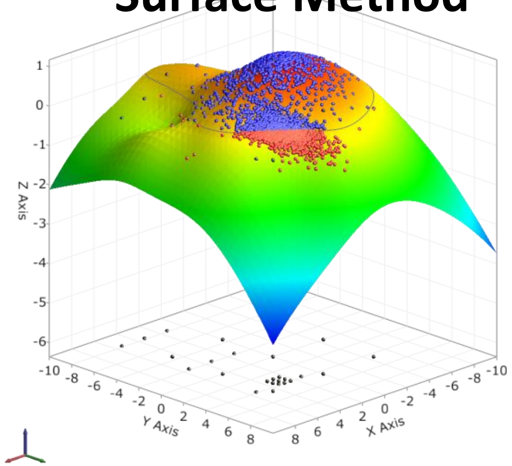


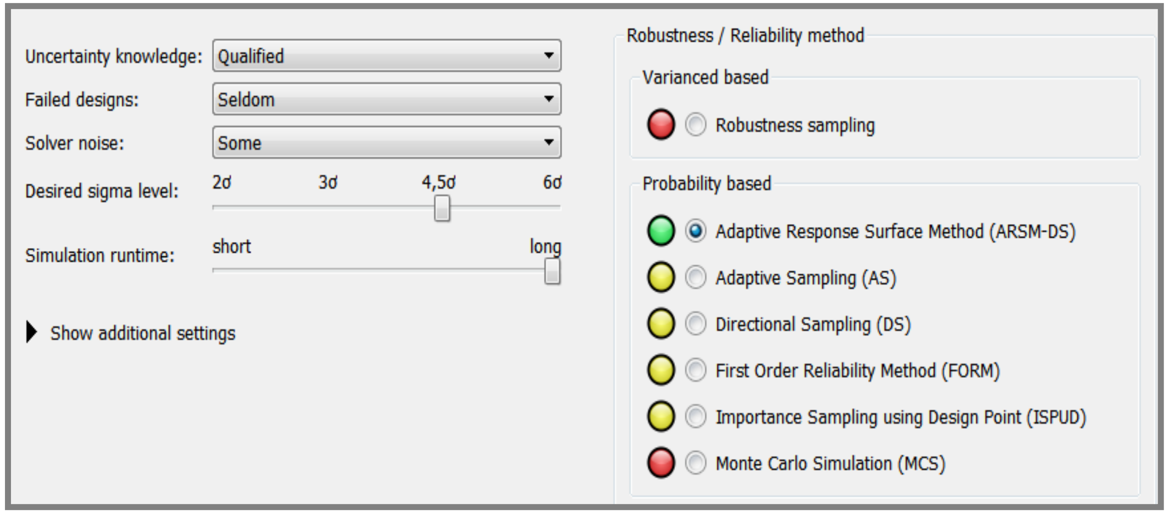
$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{h_{\mathbf{Y}}(\mathbf{x}_i)} I\left(g(\mathbf{x}_i)\right)$$

Adaptive Importance Sampling

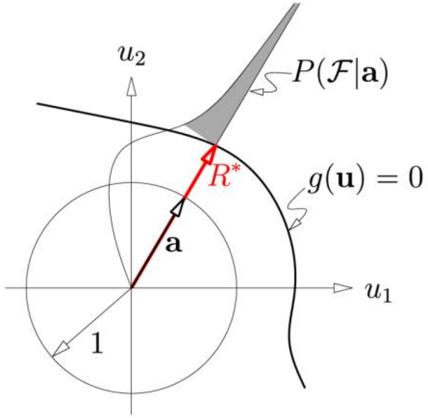


Adaptive Response Surface Method



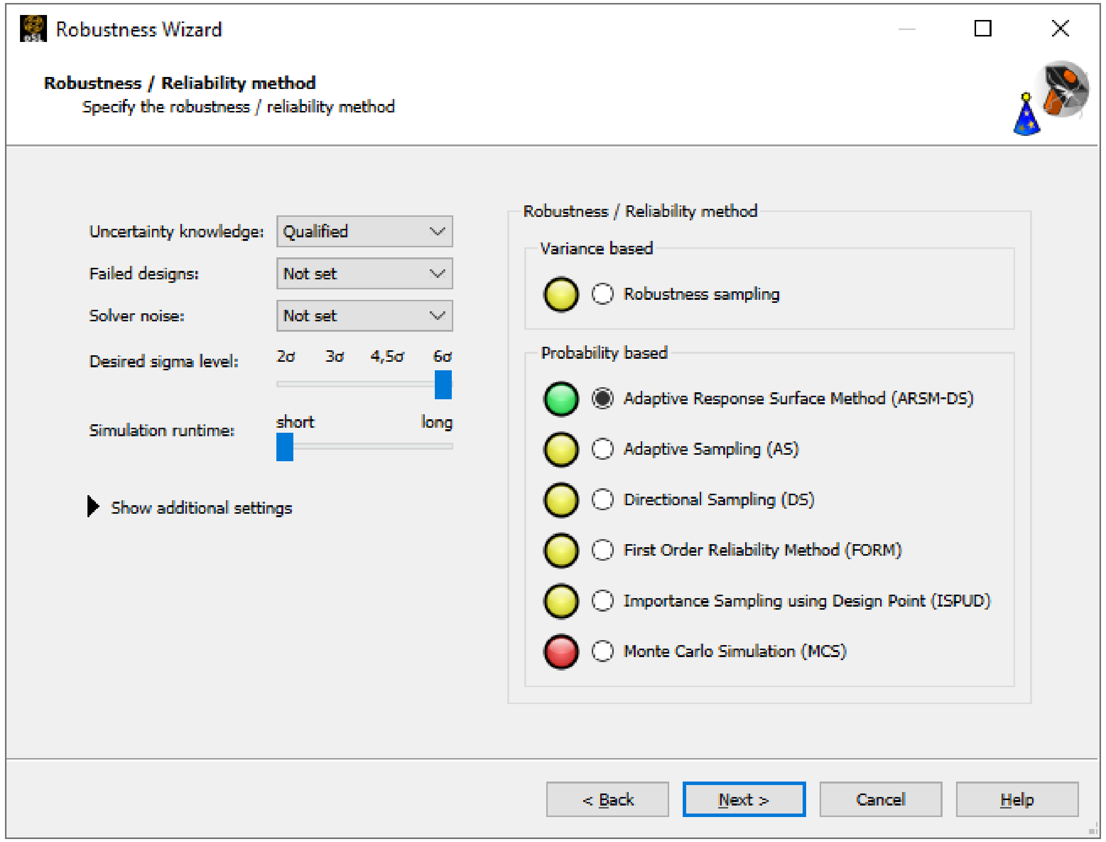


Directional Sampling



Comparison of the determined PoF

Reliability algo.	No. samples	PoF
ARSM-DS	300	6.4e-07
AS	2 000	4.7e-07
DS	1 085	5.2e-07
FORM	890	1.3e-06
FORM + ISPUD	+ 6 000	4.7e-07
MCS[1]	200 000 000 (aborted at 300 000)	?



[1] estimation Monte-Carlo-Simulation for COV=10%:

$$\mathbf{n \geq 100/PoF_{est}} \approx 100/5e\text{-}07 = 2.0e{+}08!$$

Directional Sampling

shown in standard gaussian space

PoF = 5.2e-07 ✓

Number of directions: 100

Number of parallel solver calls: 10

Line search phase

Number of parallel solver runs in presample: 10

Line search method: Bisection

Line search accuracy: 0.001

Complete directions : 100 / 100 !

Probability of Failure : 5.18627e-07

Standard deviation error : 2.39929e-07 !

Reliability Index : 4.88444

Number of designs

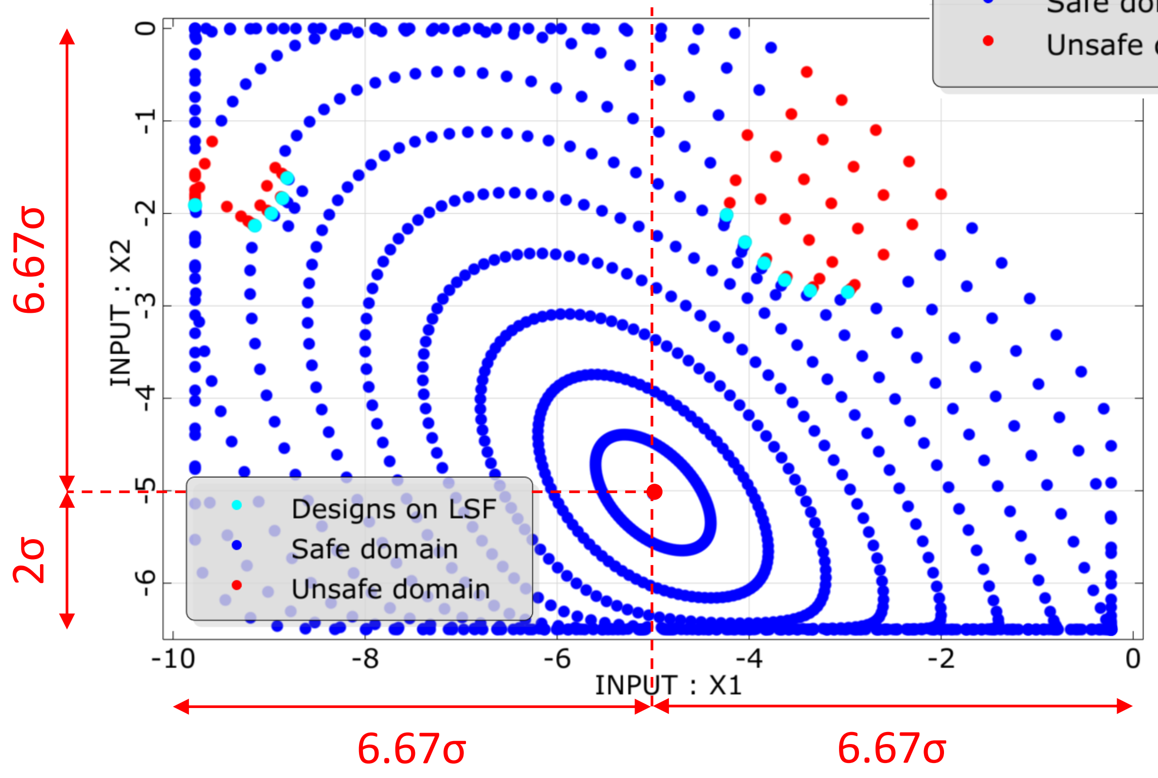
Total : 1085

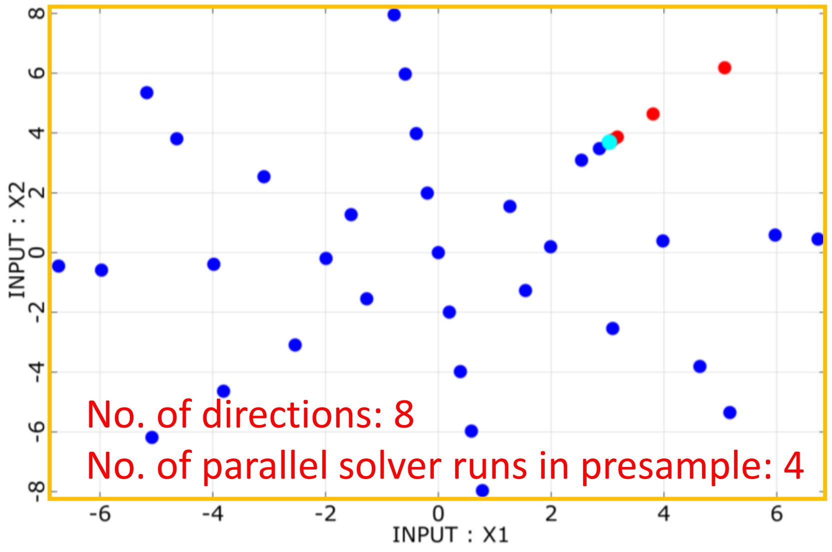
Safe domain : 1009

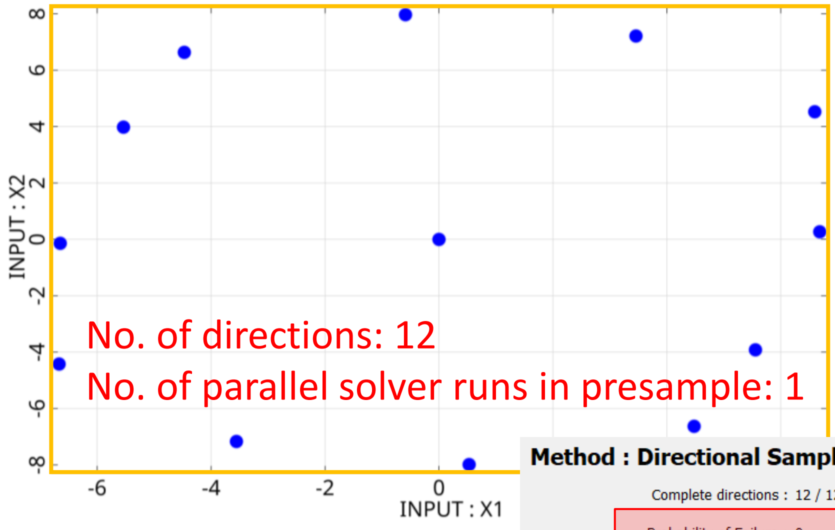
Unsafe domain : 76

Failure strings : 0

Failed : 0







Method : Directional Sampling (DS)

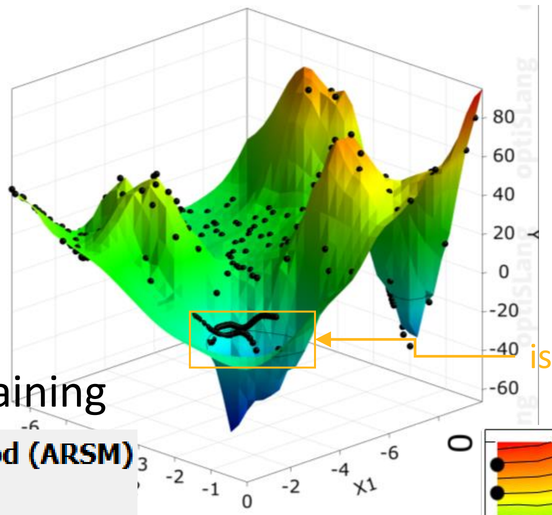
Complete directions : 12 / 12

Probability of Failure : 0

Standard deviation error : inf !

Reliability Index : 10

ARSM-DS



PoF = 6.4e-07 ✓
using 300 design for ARSM training

Metamodel	
Number of supports in 1st step:	200
Scaling factor in 1st step:	3.00
Number of supports per step:	50
Number of steps:	3

Directional Sampling	
Number of directions:	1000

Method : Adaptive Response Surface Method (ARSM)

Complete approximations : 3 / 3
Selected data : 3. Approximation

Probability of Failure : 6.4404e-07
Standard deviation error : 8.52822e-08
Reliability Index : 4.84158

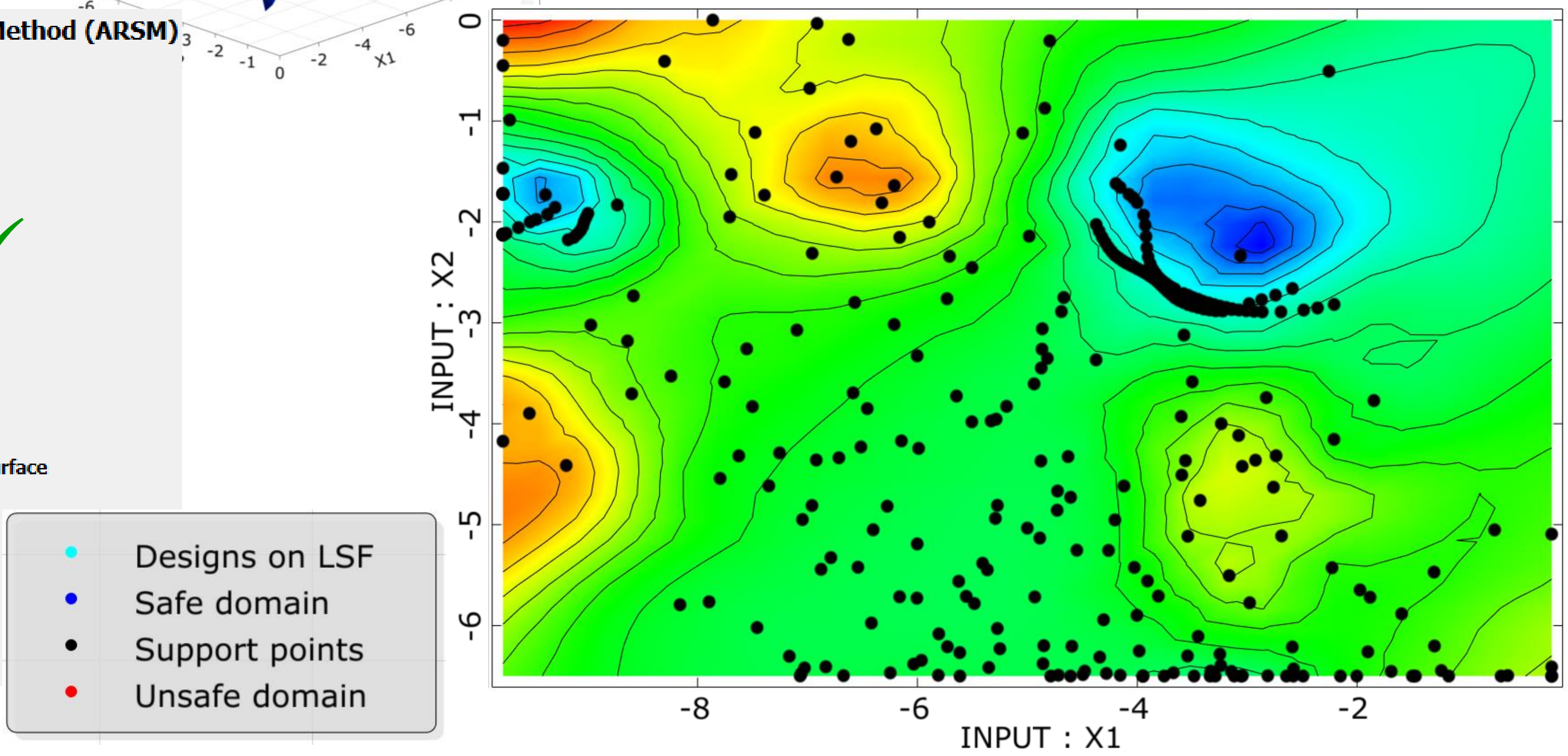
Approximation errors : R^2 / R^2pred ✓
lim_st_Y: 1 / 0.98249

Number of designs
Total : 300
Safe domain : 250
Unsafe domain : 50
Failure strings : 0
Failed : 0

Directional Sampling on Response Surface

Complete directions : 1000 / 1000

Number of designs
Total : 5865
Safe domain : 5269
Unsafe domain : 596
Failure strings : 0
Failed : 0



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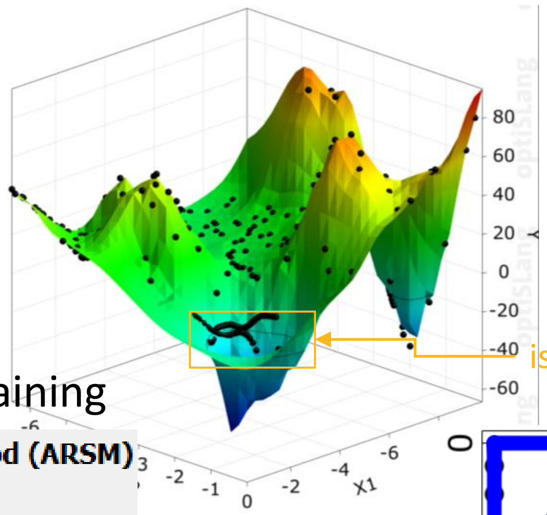
ARSM-DS

PoF = 6.4e-07 ✓

using 300 design for ARSM training



isoline Y = -20

Metamodel	
Number of supports in 1st step:	200
Scaling factor in 1st step:	3.00
Number of supports per step:	50
Number of steps:	3
Directional Sampling	
Number of directions:	1000

Method : Adaptive Response Surface Method (ARSM)

Complete approximations : 3 / 3
Selected data : 3. Approximation

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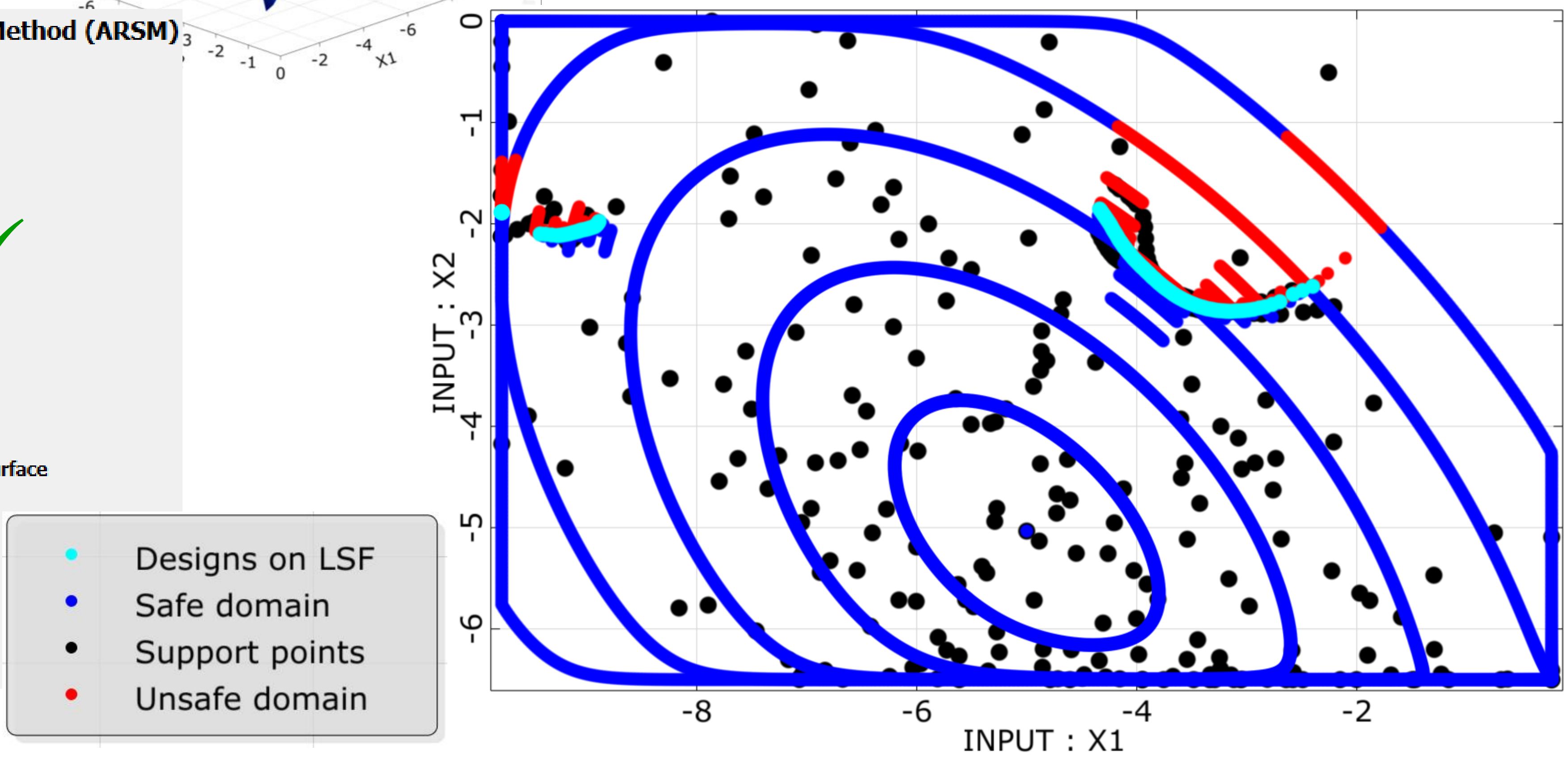

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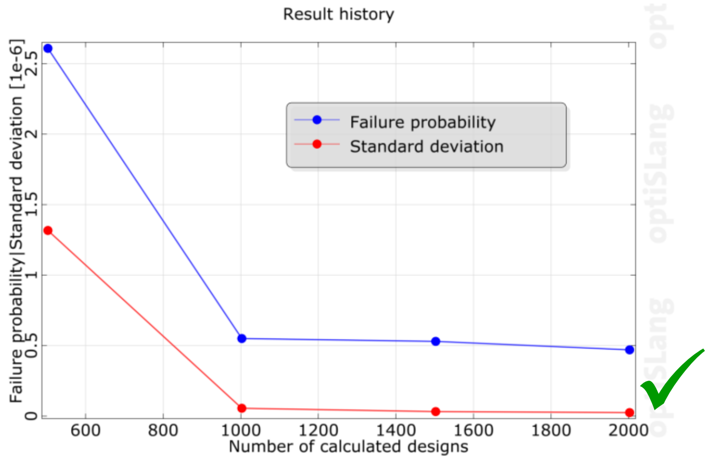
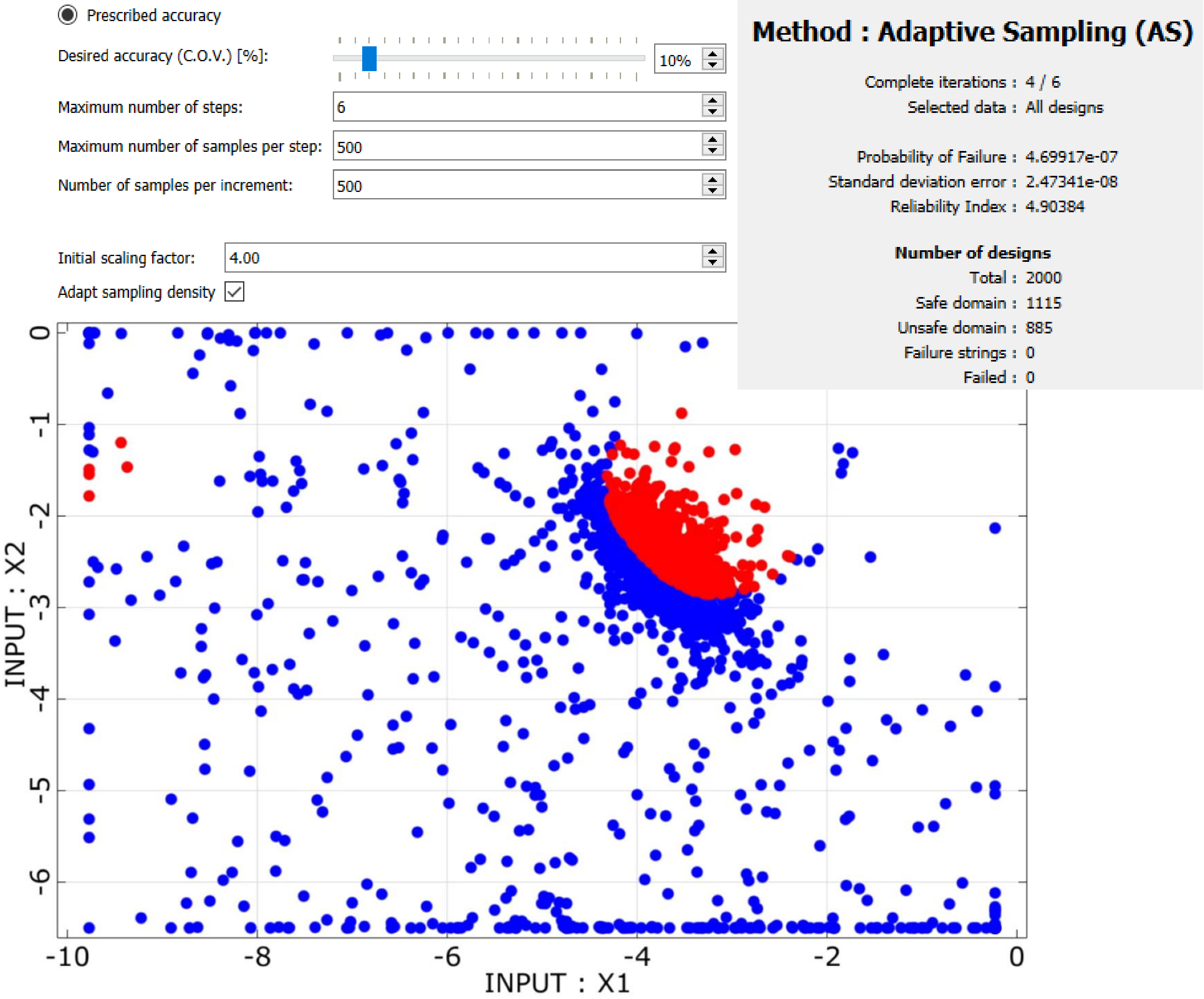
Adaptive Sampling

PoF = 4.7e-07 ✓

- in first step scan of parameter space
- statistical information about failure domain are used to increase amount of failure events
- focus on most probable failure domain
- check for converged results

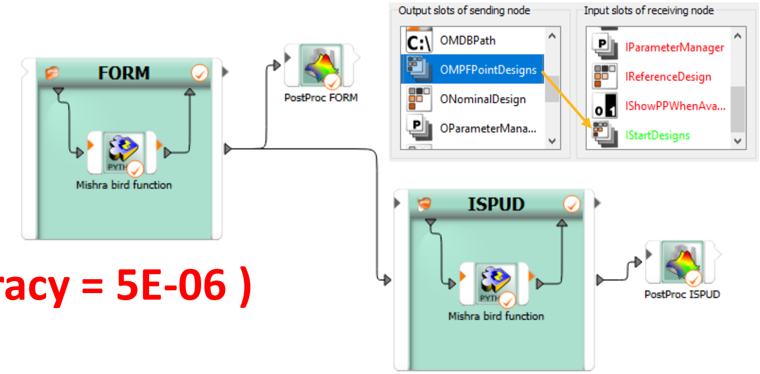


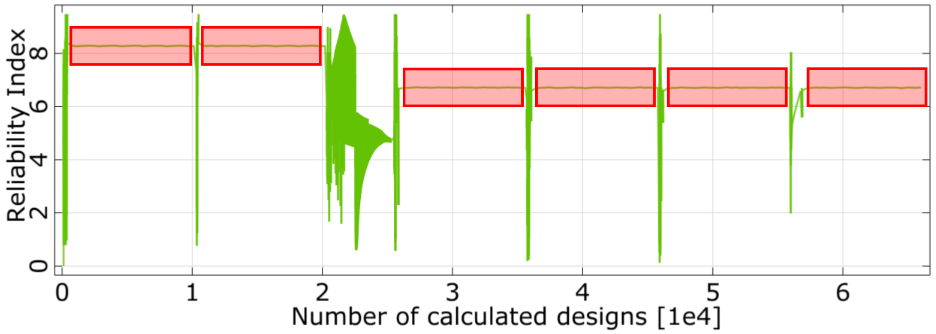


FORM (*) + ISPUD

(*) default settings
(no start designs, **desired accuracy = 5E-06**)

PoF = 1.3e-06





Method : First Order Reliability Method (FORM)

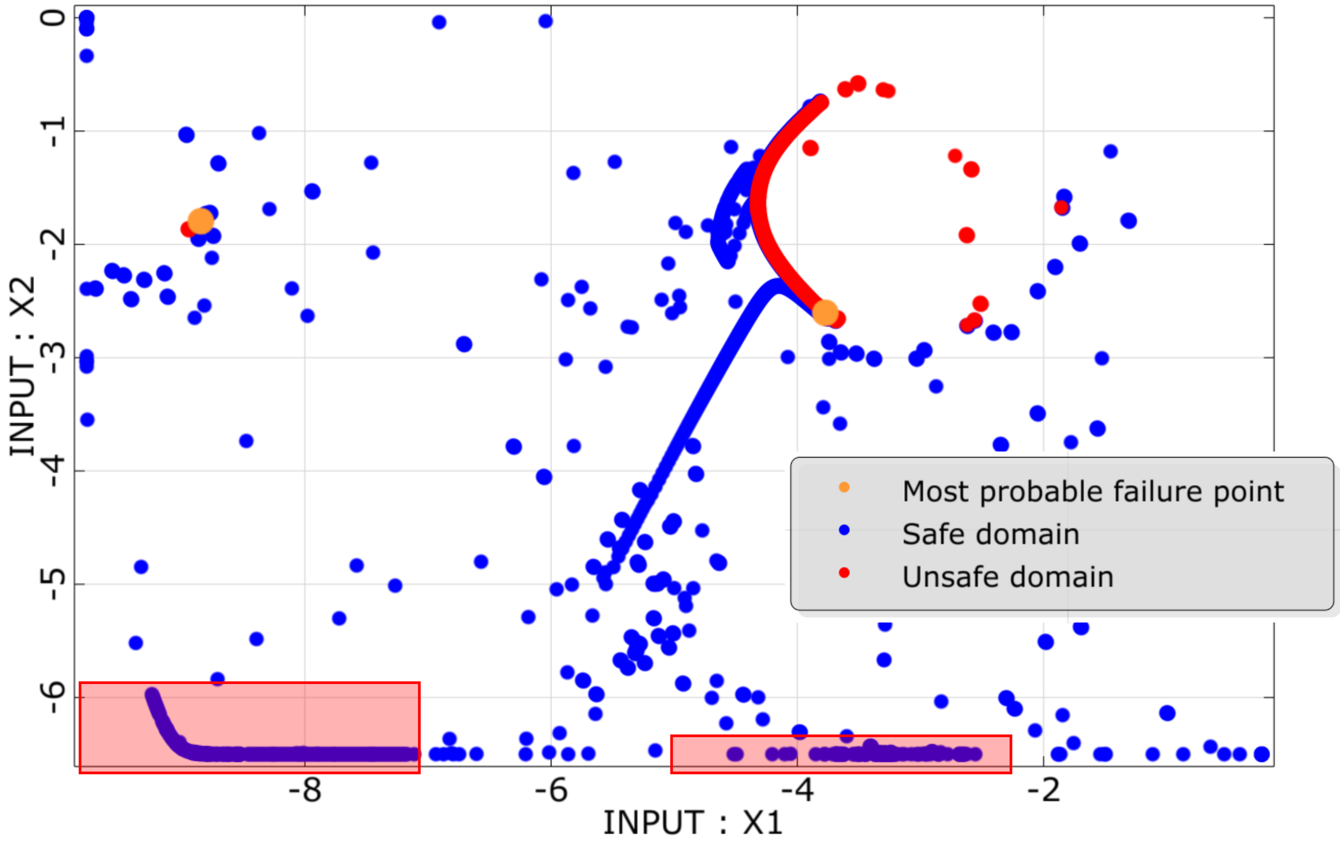
Probability of Failure : 1.3153e-06
Reliability Index : 4.69774

Number of designs

Total : 65881
Safe domain : 64634
Unsafe domain : 1247
Failure strings : 0
Failed : 0

Most probable failure point(s)

ID :	25415	20266
Input parameter values		
X1 :	-3.77261	-8.84384
X2 :	-2.60422	-1.79514
Reliability index (FORM) :	4.70378	5.37471
Probability of failure (FORM) :	1.27696e-06	3.83536e-08



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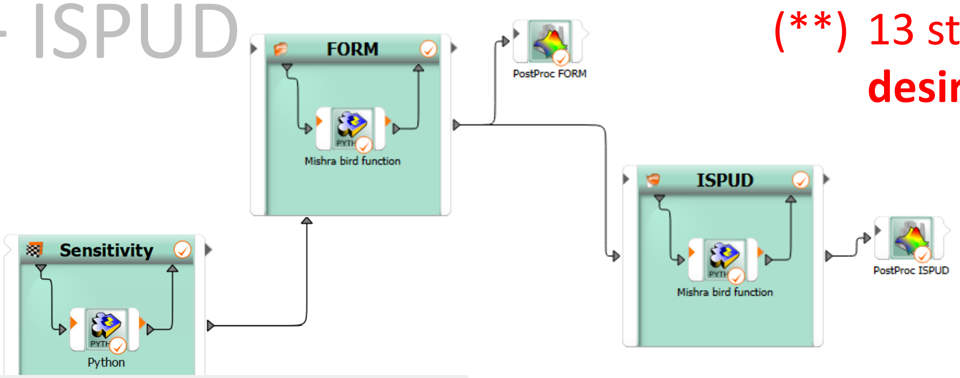
FORM (**) + ISPUD

(**) 13 start designs violating limit from sensitivity study,
desired accuracy = 5E-03

PoF = 1.3e-06



Method : First Order Reliability Method (FORM)

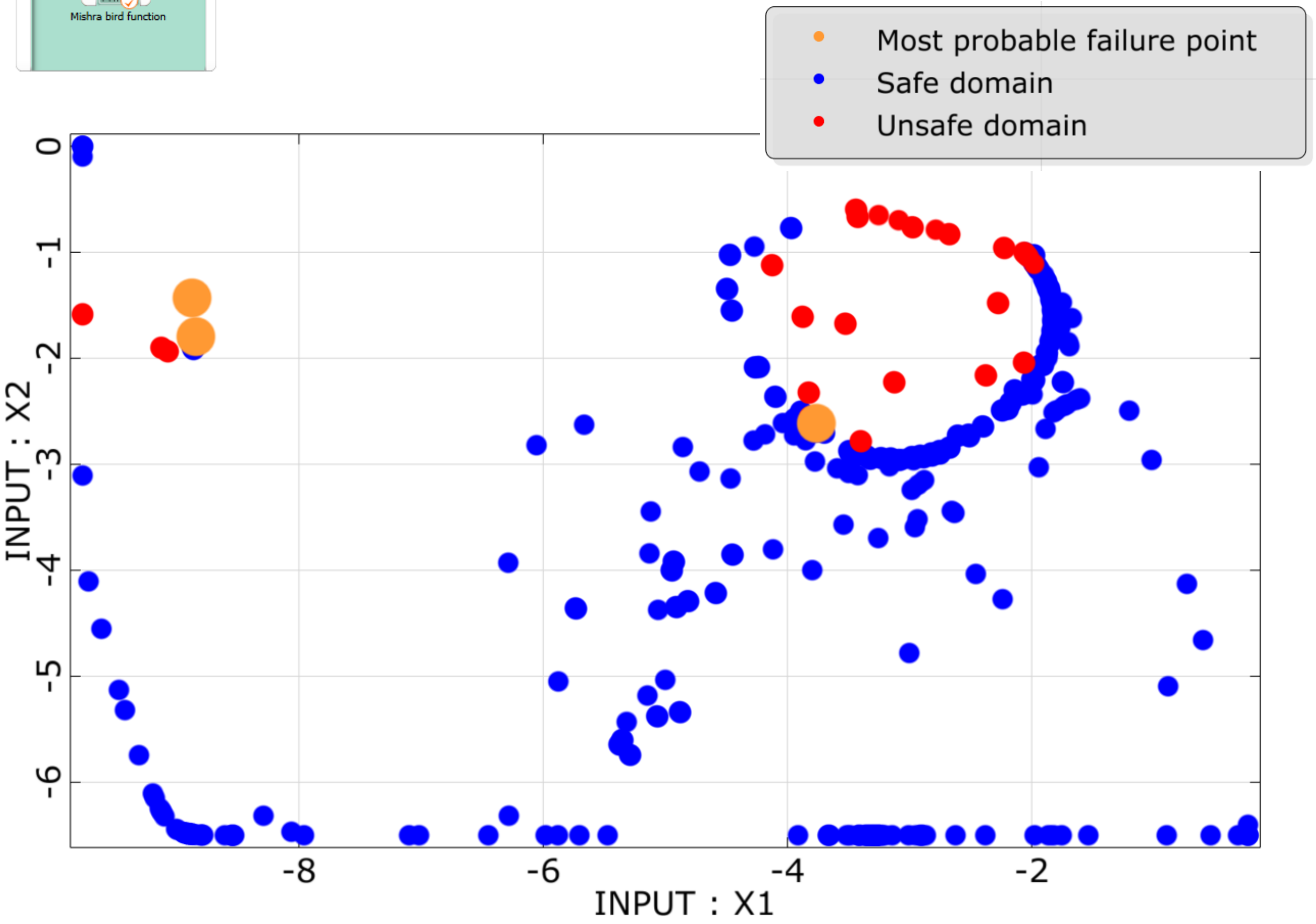
Probability of Failure : 1.31819e-06
Reliability Index : 4.69729

Number of designs

Total :	890
Safe domain :	734
Unsafe domain :	156
Failure strings :	0
Failed :	0

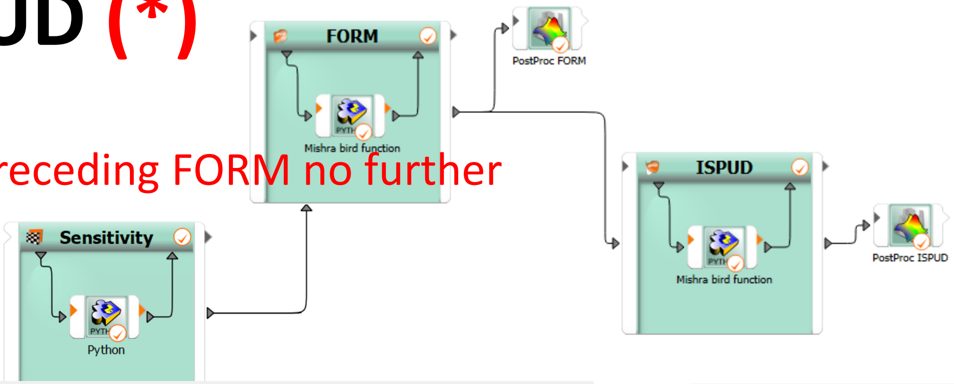
Most probable failure point(s)

ID :	406	886	455
Input parameter values			
X1 :	-3.76305	-8.84377	-8.87509
X2 :	-2.61315	-1.79501	-1.43022
Reliability index (FORM) :	4.70337	5.37468	5.58037
Probability of failure (FORM) :	1.2795e-06	3.83591e-08	1.20003e-08



FORM + ISPUD (*)

(*) using 3 MOPF from preceding FORM no further search



PoF = 4.7e-07 ✓

Method : Importance Sampling Procedure Using Design Points (ISPUD)

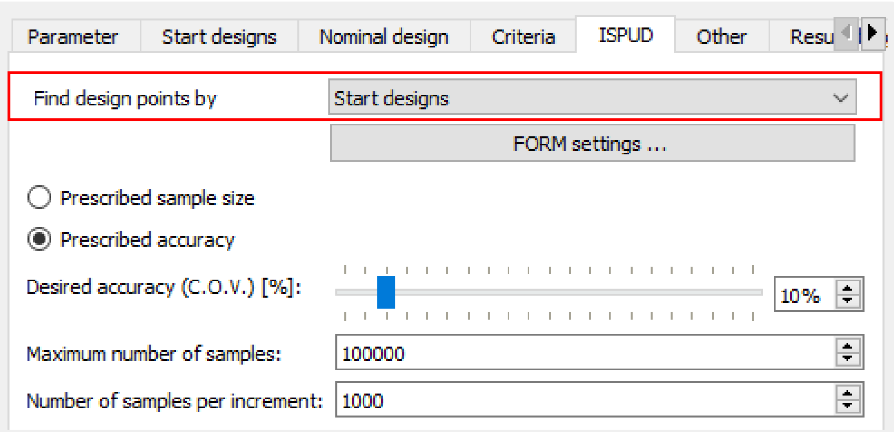
Selected data : All designs

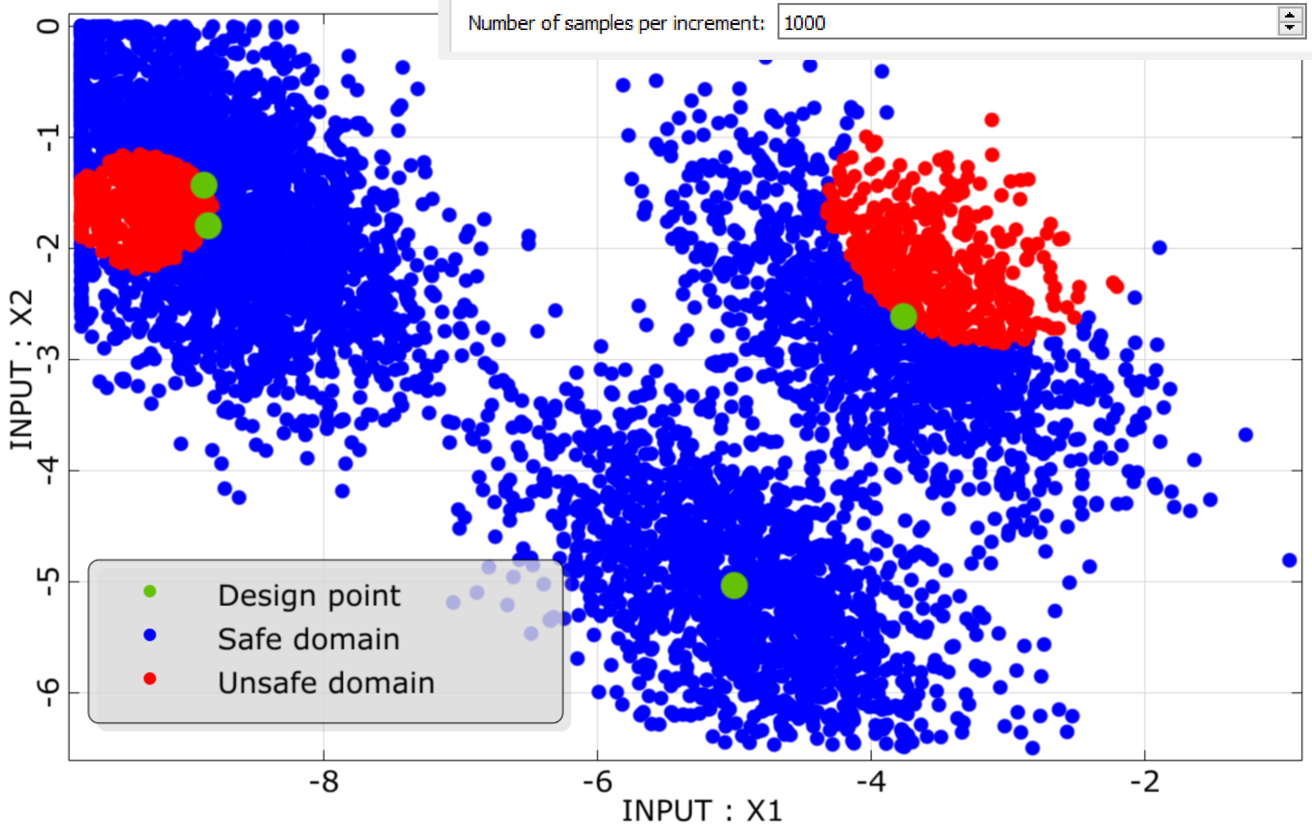
Probability of Failure : 4.73743e-07
Standard deviation error : 4.29418e-08
Reliability Index : 4.90224

Number of designs
Total : 6004
Safe domain : 5045
Unsafe domain : 959
Failure strings : 0
Failed : 0

Design point(s)

ID :	2	4	3	1
Input parameter values				
X1 :	-3.76305	-8.84377	-8.87509	-5
X2 :	-2.61315	-1.79501	-1.43022	-5.03368
Reliability index (ISPUD) :	4.90814	5.66761	5.67859	10
Probability of failure (ISPUD) :	4.59713e-07	7.2402e-09	6.79039e-09	0





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Conclusion

- advanced reliability methods are recommended for probability < 1/1000 since effort for Monte-Carlo approach increases inversely proportional to expected probability
- before reliability analysis run sensitivity study within the bounds of stochastic parameters or a robustness analysis to gain deeper design understanding
- reliability analysis operate in Standard Normal/Gaussian Space
- reliability analysis in two steps: fast detection of failure mechanisms and efficient quantification of failure probability
- failure mechanisms detection: scan of the stochastically space up to 8 Sigma or by scaling the standard deviation (3.0 by default)
- failure mechanisms detection is easy to use for half-space with transition from safe to unsafe domain, but search need to be adapted for local spots

Ansys

Probability of failure for the cut-out scenario

Scenario-Based Safety Assessment using Ansys optiSLang

Customer Goals

Milage required to proof AD/ADAS system safety cannot be tested operational in field ➔ scenarios need to be simulated

Simulating complete required milage is also not feasible ➔ collision relevant scenarios need to be identified to significantly reduce number of required simulations

Solution

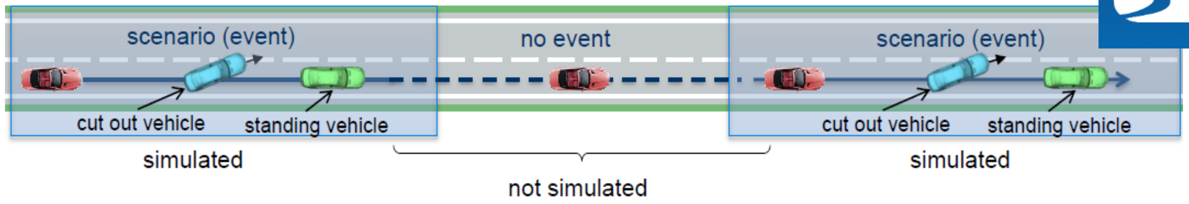
Only «interesting» logical scenarios are analyzed

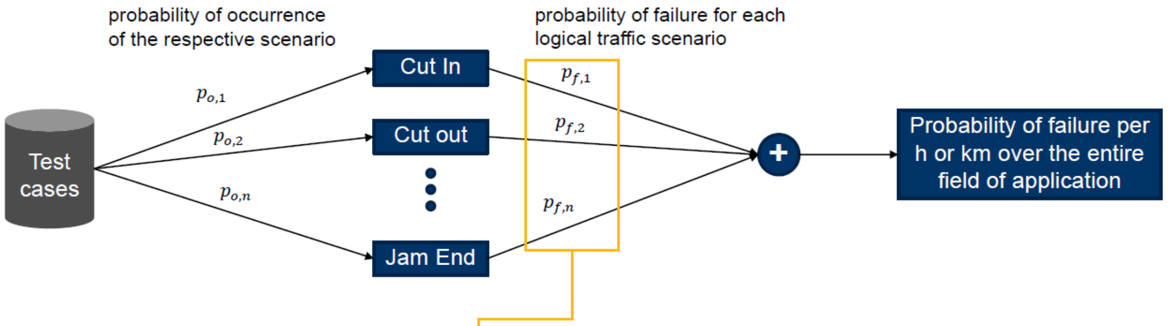
Sensitivity analysis in optiSLang allows for the identification of parameters with highest impact & model failure

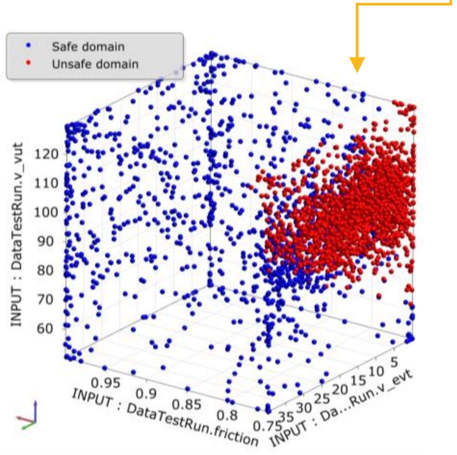
Reliability analysis in optiSLang to determine the probability of failure for a logical scenario to compare performance between ADAS software versions & identification of critical parameters

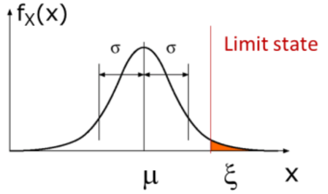
Benefits

- Allows for ADAS software function testing, verification & certification
- Identification of critical / relevant parameters
- **Number of simulation scenarios can be reduced by factor 1000**









Probability of Failure : 0.0454044
Standard deviation error : 0.0013054
Reliability Index : 1.69115

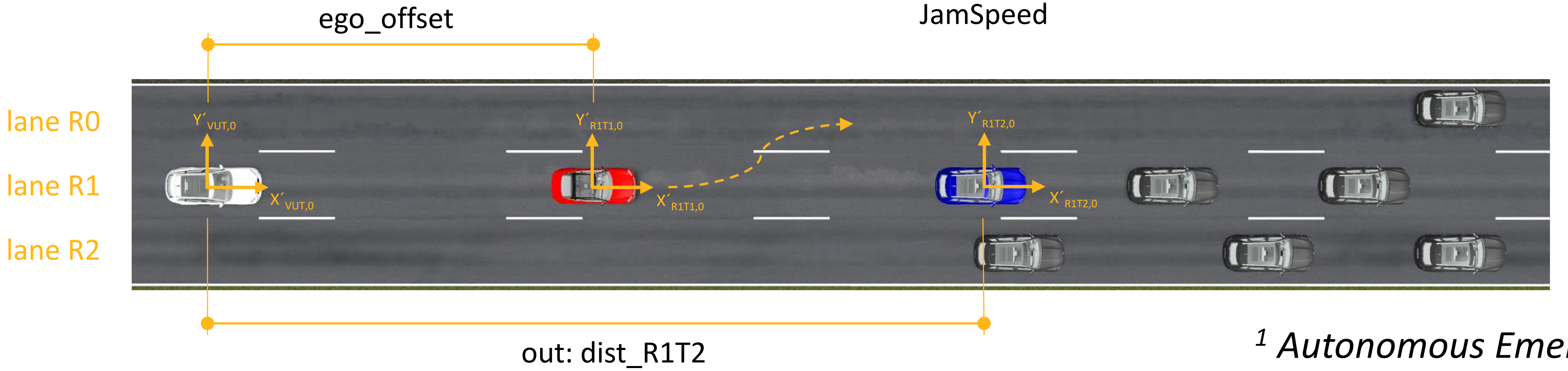
Number of designs
Total : 3000

$$P(\text{crash}/\text{km}) = P(\text{crash} \mid \text{scenario}_1) P(\text{scenario}_1/\text{km}) + \cdots + P(\text{crash} \mid \text{scenario}_{rest}) P(\text{scenario}_{rest}/\text{km})$$

Cut-out scenario simulation

- ideal sensors to measure distance to preceding car
- custom AEB[1] function by FMU-Plug-In
- emergency braking initiation based on:
  Time-to-Collision (TTC) and Time-to-break-threshold (TTBT)

VUT	**R1T1**	**JAM**
(Vehicle Under Test)	(lane R1 Target 1)	
VUT_PosX0, VUT_PosY0 VUT_Speed	R1T1_PosX0, R1T1 _PosY0 R1T1 _Deceleration R1T1 _Speed	PosY0_R0 PosY0_R1 PosY0_R2 JamSpeed



[1] *Autonomous Emergency Braking (AEB)*

Parameter definition

11 parameters used for sensitivity/optimization

9 parameters used for stochastic analyses

4 dependent parameters

	Name	Parameter type	Reference value	Value type	Resolution	Range		Range plot	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	Plugins.MaxDecel	Optimization	4.7	REAL	Continuous	4	9							
2	DataTestRun.duration_s1	Optimization	3	REAL	Continuous	2.7	3.3							
3	DataTestRun.R1T1_PosY0	Opt.+Stoch.	0	REAL	Continuous	-1	1			TRUNCATEDNORMAL	0	0.328567	100 %	0; 0.333; -1; 1
4	DataTestRun.R1T1_Speed	Opt.+Stoch.	67.5	REAL	Continuous	35	100			UNIFORM	67.5	18.7639	27.7983 %	35; 100
5	DataTestRun.Jam_PosY0_R0	Opt.+Stoch.	4	REAL	Continuous	3	5			TRUNCATEDNORMAL	4	0.328567	8.21417 %	4; 0.333; 3; 5
6	DataTestRun.Jam_PosY0_R1	Opt.+Stoch.	0	REAL	Continuous	-1	1			TRUNCATEDNORMAL	0	0.328567	100 %	0; 0.333; -1; 1
7	DataTestRun.Jam_PosY0_R2	Opt.+Stoch.	-3.5	REAL	Continuous	-4.5	-2.5			TRUNCATEDNORMAL	-3.5	0.328567	9.38762 %	-3.5; 0.333; -4.5; -2.5
8	DataTestRun.Jam_Speed	Opt.+Stoch.	20	REAL	Continuous	0	30			TRUNCATEDNORMAL	20	5	25 %	20.514; 5.49591; 0; 30
9	DataTestRun.VUT_PosY0	Opt.+Stoch.	0	REAL	Continuous	-1	1			TRUNCATEDNORMAL	0	0.328567	100 %	0; 0.333; -1; 1
10	R1T1_Deceleration	Opt.+Stoch.	6	REAL	Continuous	1	10			TRIANGULAR	6	1.87083	31.1805 %	1; 7; 10
11	ego_offset	Opt.+Stoch.	49.312	REAL	Continuous	44.3808	54.2432			TRUNCATEDNORMAL	49.312	21.3809	43.3584 %	34.7109; 30.229; 18.037; 300

	Name	Parameter type	Reference value	Operation
12	DataTestRun.R1T1_PosX0	Dependent	1186.25	1210.0-(DataTestRun.R1T1_Speed-DataTestRun.Jam_Speed)/2
13	DataTestRun.VUT_PosX0	Dependent	1136.94	DataTestRun.R1T1_PosX0-ego_offset
14	DataTestRun.VUT_Speed	Dependent	67.5	DataTestRun.R1T1_Speed
15	DataTestRun.R1T1_Speed_s1	Dependent	49.5	DataTestRun.R1T1_Speed-DataTestRun.duration_s1*R1T1_Deceleration

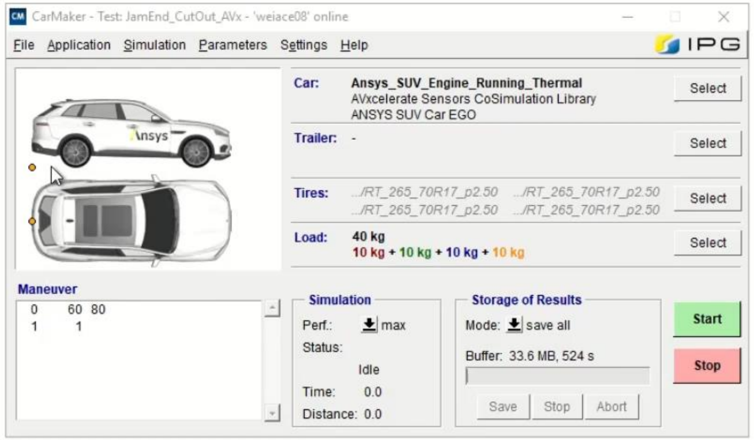
Define parameter correlations

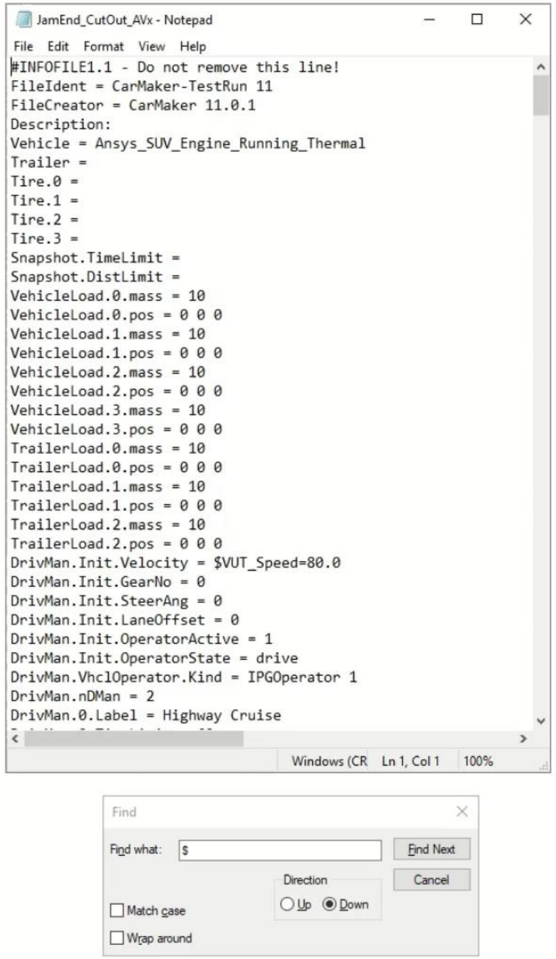
	DataTestRun.R1T1_Speed	ego_offset
ego_offset	0.65	1
DataTestRun.R1T1_Speed	1	0.65

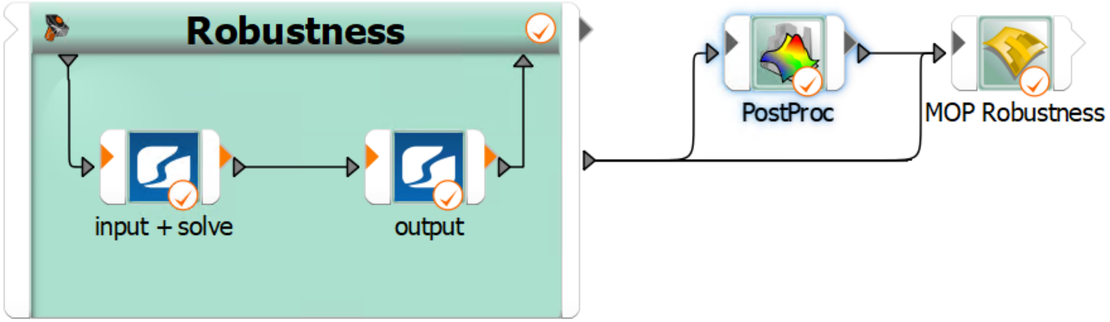
Restore Defaults | OK | Cancel | Apply

Ansys

Set up Parametric Variation Analysis

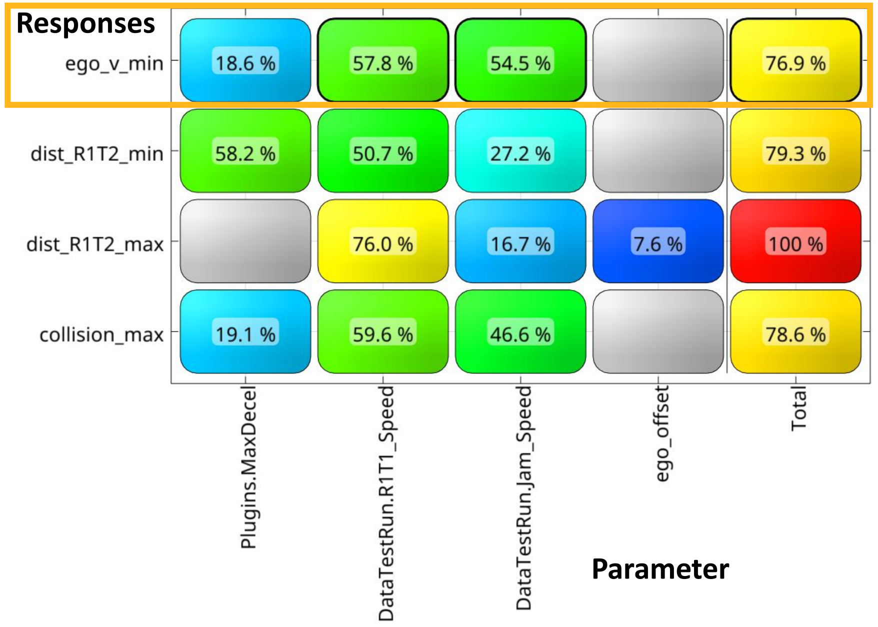


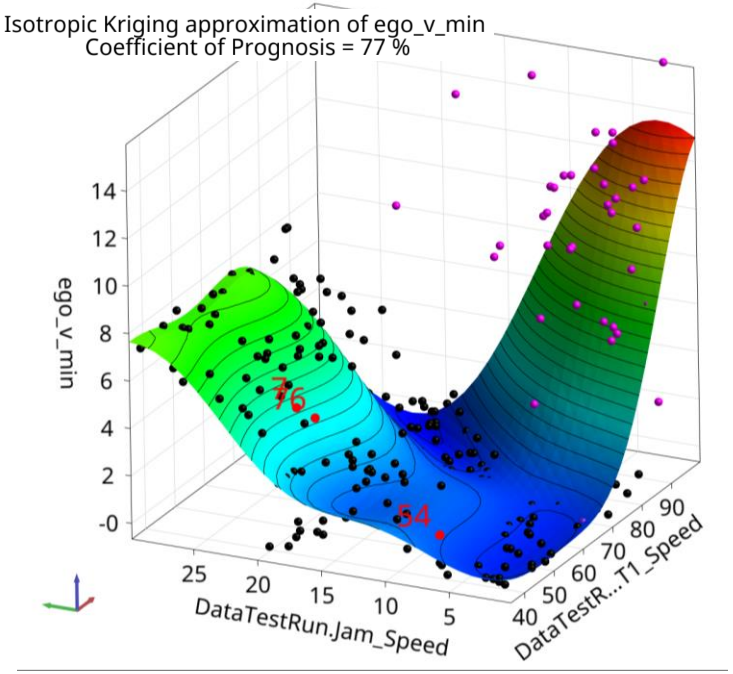


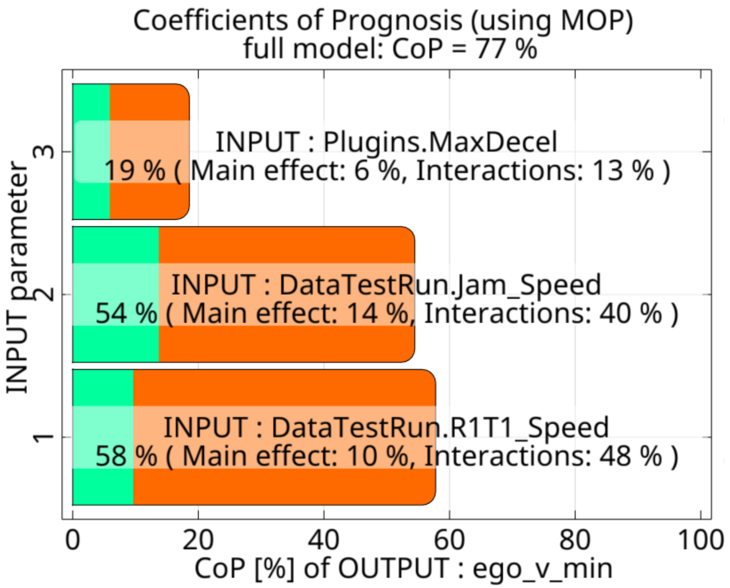


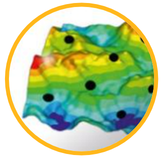
Sensitivity Analysis

- scanning space of 11 parameters of type ‘Optimization’
- approximation by surrogate model without over-fitting, objective measure of prognosis quality = **CoP**
- automatic determination of relevant parameter subspace









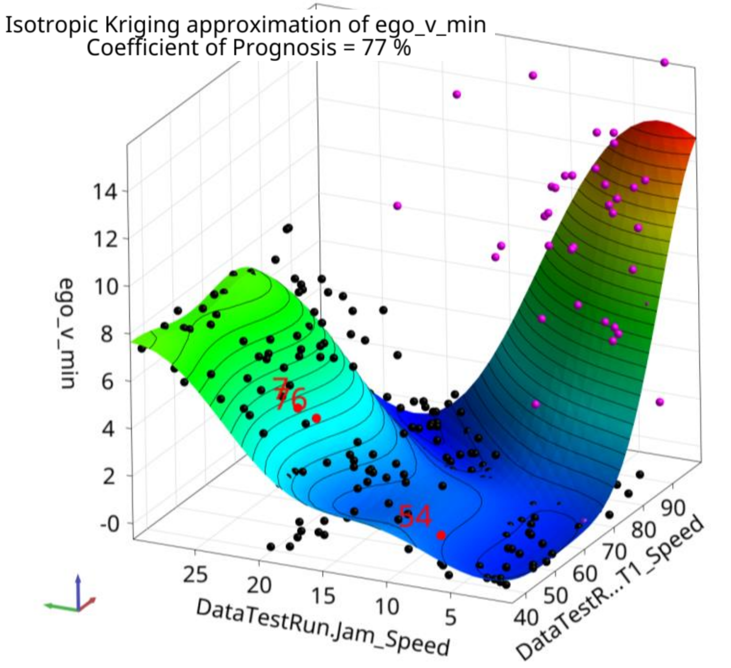
Sensitivity Analysis

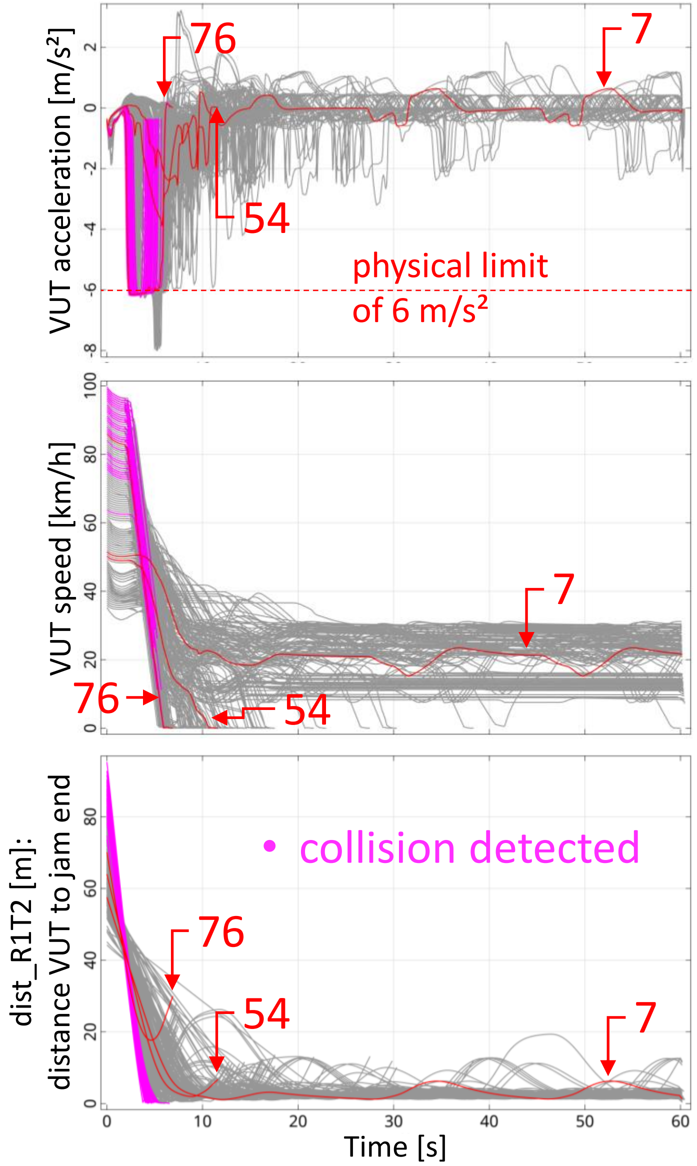
From **VUT speed** control by the AEB function can be observed:

- if situation permits, VUT will decelerate to jam speed and follows at safe distance (e.g. 7)
- braking to zero is forced only in hazardous situations (eg. 76, 54)
- physical limitation of VUT deceleration to 6 m/s²

Responses	Plugins.MaxDecel	DataTestRun.R1T1_Speed	DataTestRun.Jam_Speed	ego_offset	Total
ego_v_min	18.6 %	57.8 %	54.5 %		76.9 %
dist_R1T2_min	58.2 %	50.7 %	27.2 %		79.3 %
dist_R1T2_max		76.0 %	16.7 %	7.6 %	100 %
collision_max	19.1 %	59.6 %	46.6 %		78.6 %

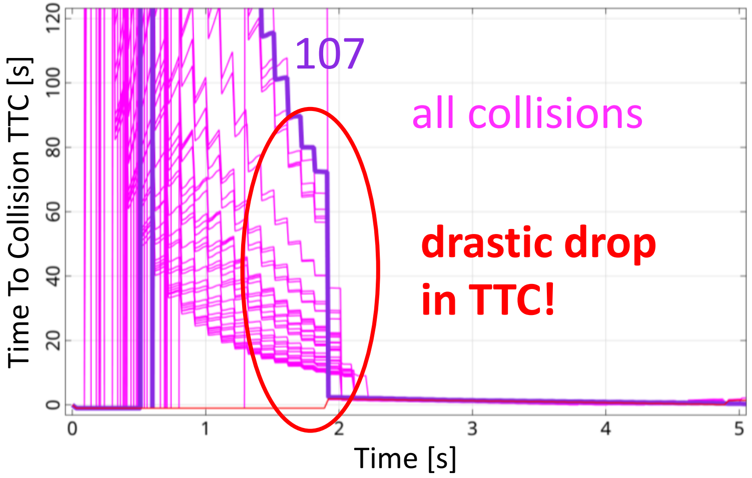
Parameter



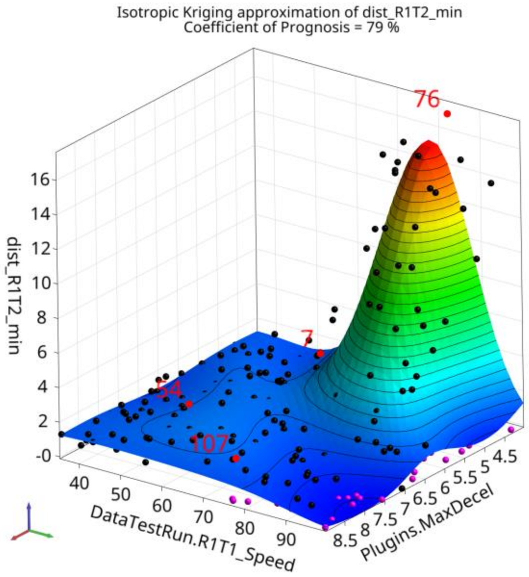


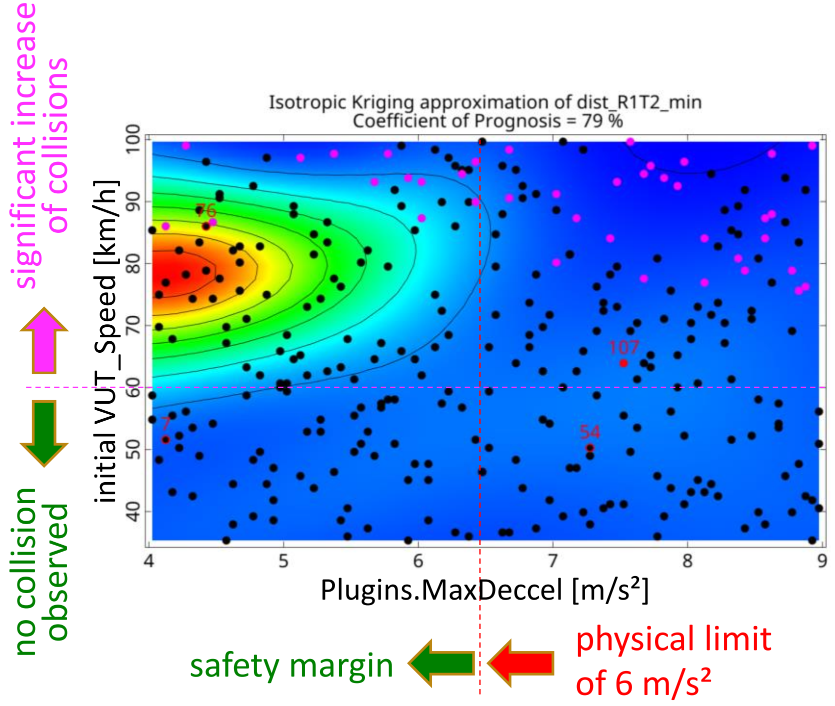
Sensitivity Analysis

- restriction to physical deceleration limit of 6m/s² needed, but for small MaxDecel min. distance is too large (e.g. 76)!
- significant increase of collisions for speed higher than 60 km/h
- ideal sensor not able to detect jam end through preceding car therefore, drastic drop in TTC => collision cannot be prevented



Responses \ Parameter	Plugins.MaxDecel	DataTestRun.R1T1_Speed	DataTestRun.Jam_Speed	ego_offset	Total
ego_v_min	18.6 %	57.8 %	54.5 %		76.9 %
dist_R1T2_min	58.2 %	50.7 %	27.2 %		79.3 %
dist_R1T2_max		76.0 %	16.7 %	7.6 %	100 %
collision_max	19.1 %	59.6 %	46.6 %		78.6 %





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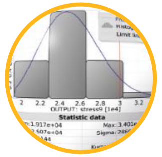
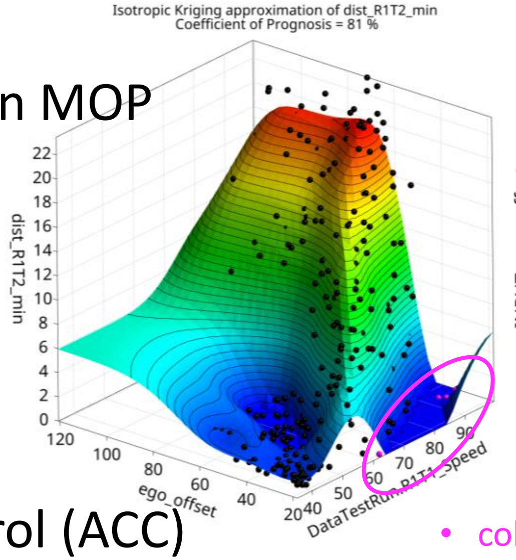


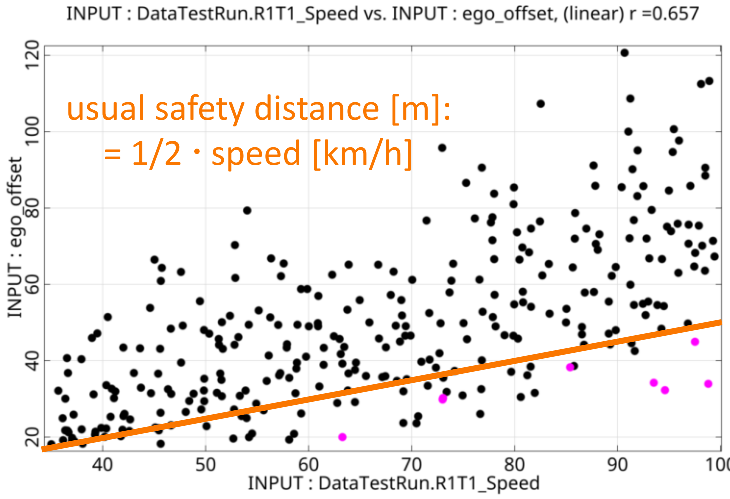
Robustness Analysis

- 9 parameters included in stochastic analyses
- automatic ranking of parameter scatter based on MOP
- statistical analysis of responses
  => probability of collision is P_rel = 3%

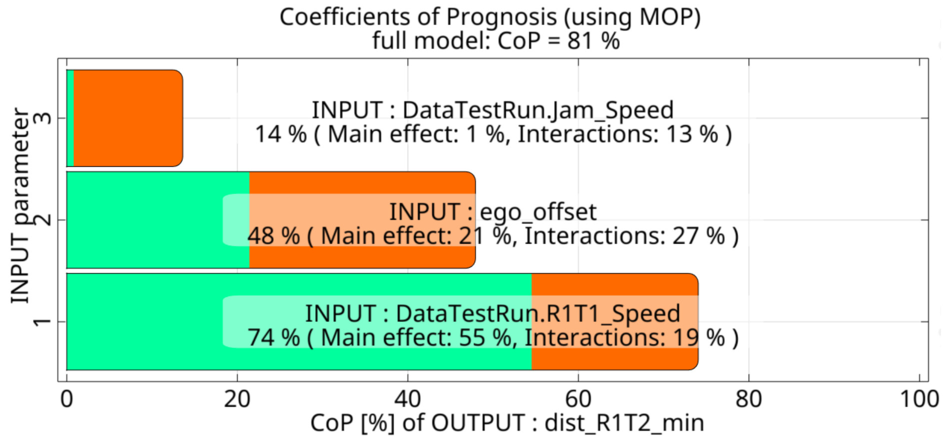
Reasons for high PoF:

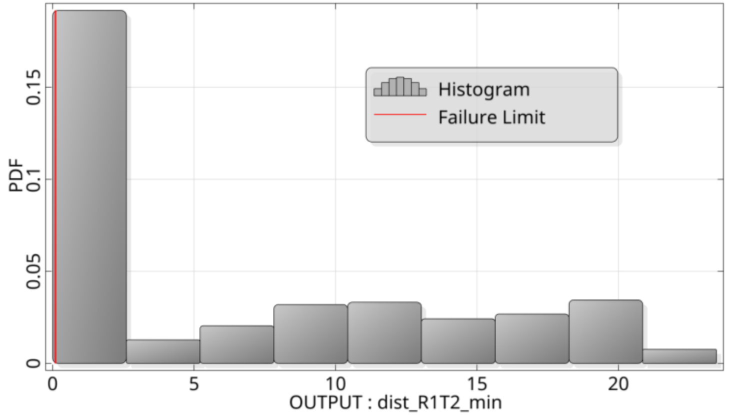
- no detection of jam end through leading car
- statistical model speed vs. offset includes speeders and pushers, no Adaptive Cruise Control (ACC)



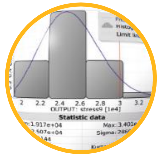


• collision detected

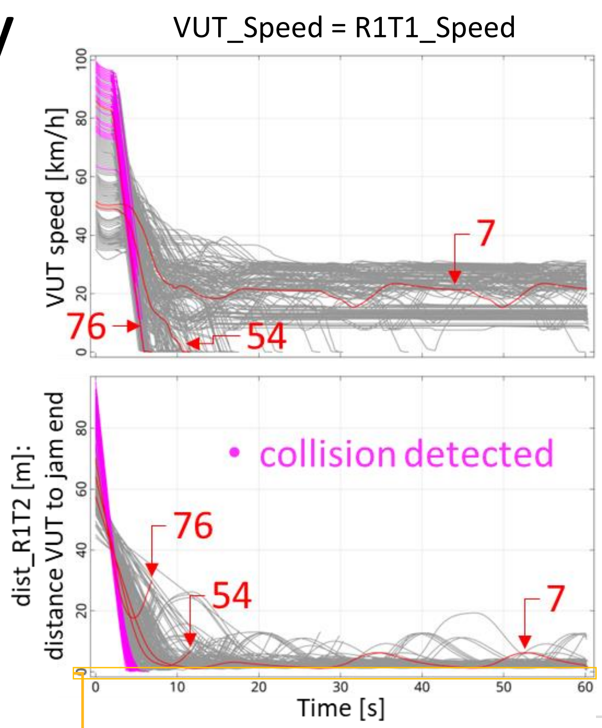




Statistical data			
Min:	4.44533e-05	Max:	23.4636
Mean value:	7.10553	Standard deviation:	7.0234
CoV:	0.98844		
Skewness:	0.693298	Excess kurtosis:	-1.00017
Limit : Failure Limit			
	Lower value = 0.1	Upper value = not set	Total
P_rel:	0.0266667		0.0266667
Sigma-Level:	0.997457		



Reliability Analyses: Limiting the impact velocity



- motivation: reliability evaluation against fatalities at high speeds, injuries at medium speeds and whiplash at low speeds
- goal: multilinear curve as limit state for impact velocity depending on initial speed
- solution: for reliability analysis mathematical expressions can be used on both sides of the limit state definition

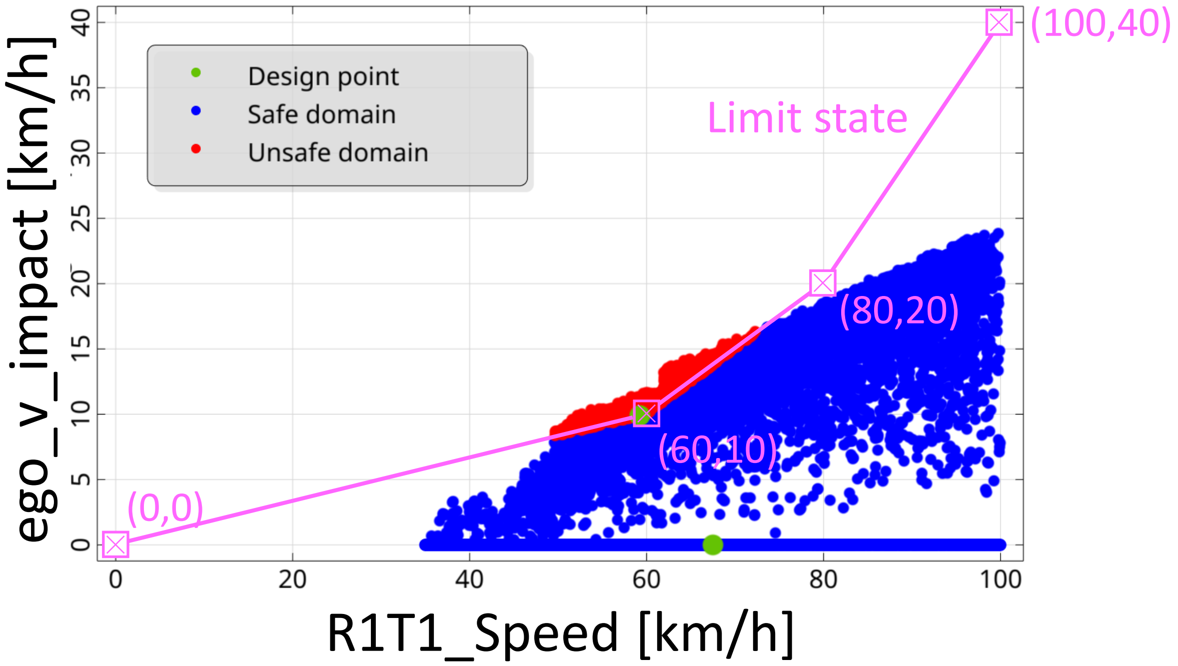
Criteria

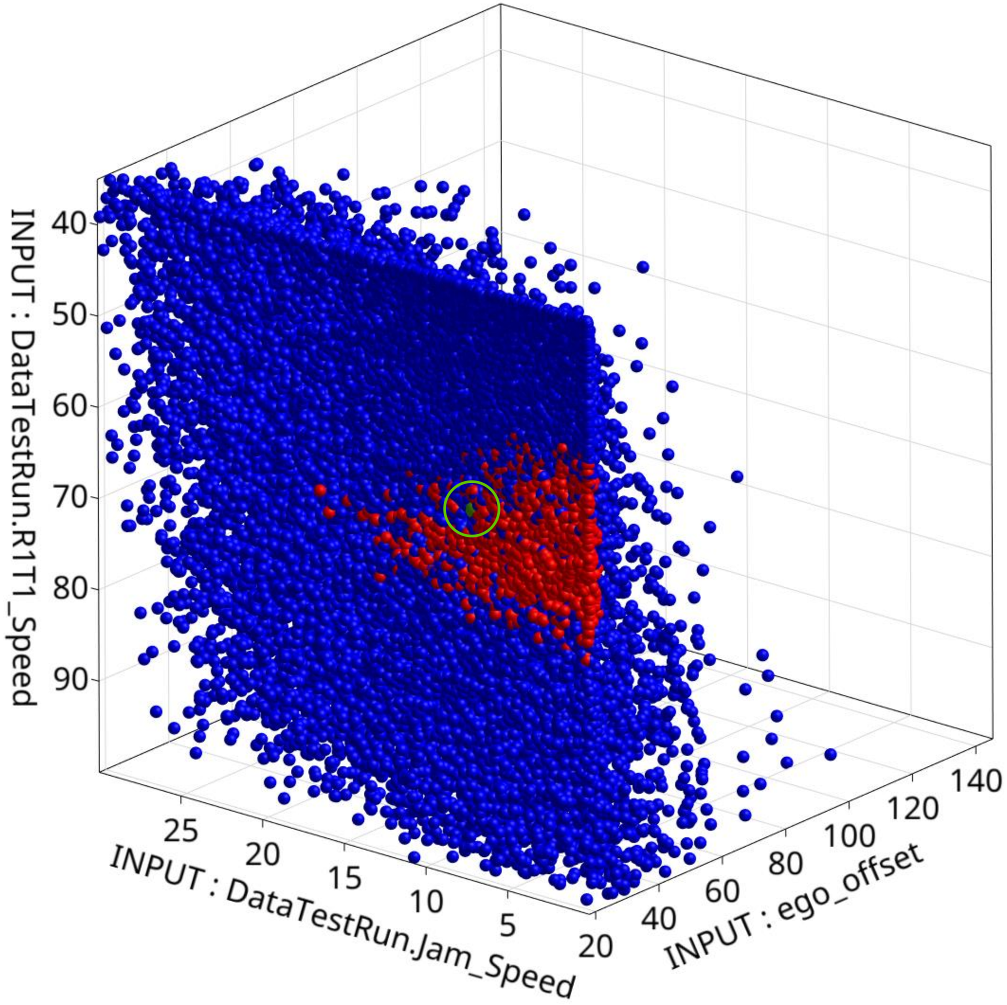
Name	Type	Expression	Criterion	Limit	Evaluated expression
limit_state_def	Limit state	ego_v_impact	≤	limit_val	0 ≤ 13.75
limit_val	Variable	extract(interpolate(signal([0,60,80,100],[0,10,20,40]),[v0],LINEAR,0,0),1)[0]			13.75
ego_v_impact	Variable	#IF dist_R1T2_min >0.1 #THEN 0 #ELSE ego_v_min			0
v0	Variable	DataTestRun.R1T1_Speed			67.5

Reliability Analyses: Limiting the impact velocity

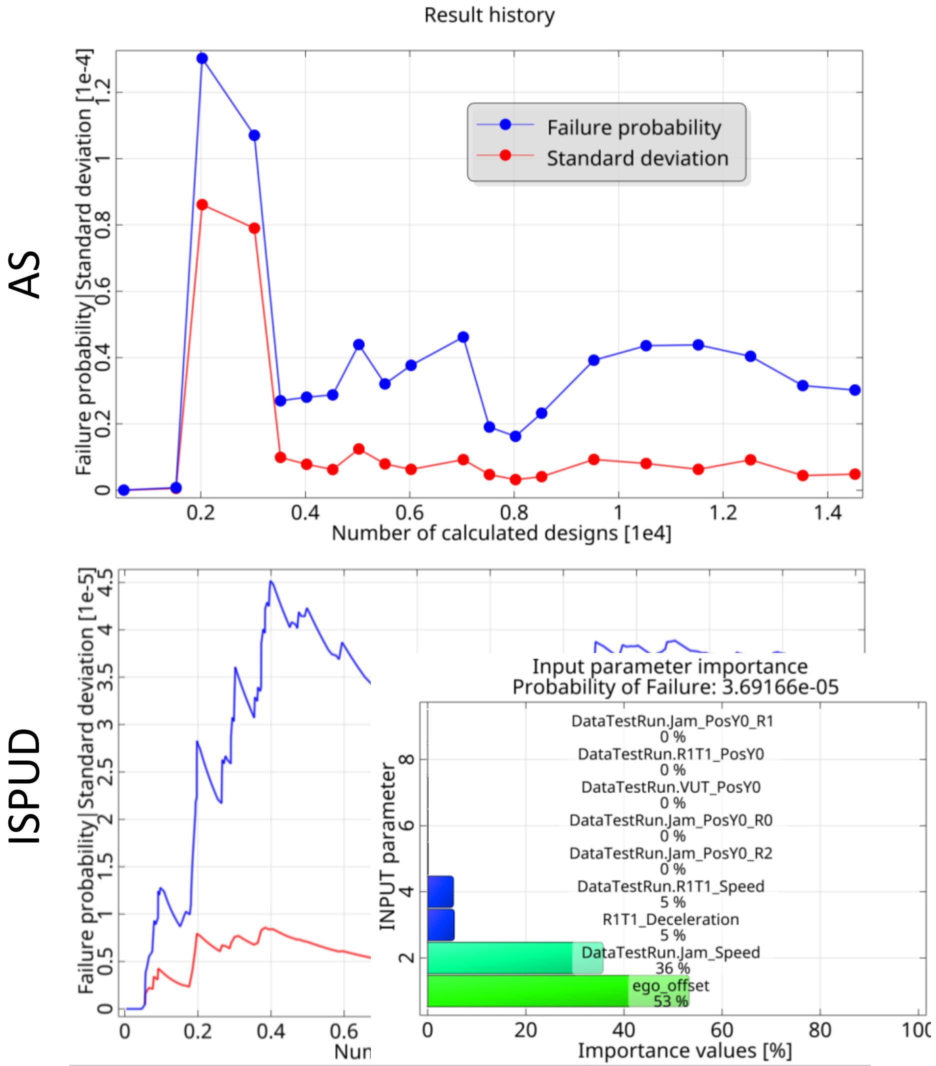
- exemplary shown results from the ISPUD analysis
- one unsafe domain with MPFPointDesign ●

R1T1_Speed	Jam_Speed	ego_offset
59.2 km/h	7.04 km/h	19.38 m





Reliability Analyses: Limiting the impact velocity



Estimation the probability of exceeding the limit state function by means of:

- Adaptive Importance Sampling (**AS**) and
- Importance Sampling Procedure using Design Points (**ISPUD**)

PoF = 3 / 100 000

Algo.	no. samples	Iteration, Inkrements	PoF	Std.Dev.	CoV [%]
AS	14 500	20 / 20	**3.02E-05**	4.84E-06	16.0%
ISPUD	20 002	20 x 1000	**3.69E-05**	4.54E-06	12.3%

MC for COV=10%: **n ≥ 100/3.3e-05** ≈ 3 Million samples

Conclusion

- scenario-based software in the loop testing of ADAS / AD functions
- automated workflows enable automated tests running overnight, e.g. after update of assistance function, of scenario-based simulation or of statistical models
- sensitivity study to scan ODD for collision-relevant scenario characteristics (edge and corner case identification)
- robustness analysis to estimate results/KPI variation and to identify safety critical inputs
- reliability analysis to efficiently quantify small probability for limit values, complex mathematical expressions can be used as limit states

Nested Robustness Evaluation in Multi-objective Design Optimization Validation

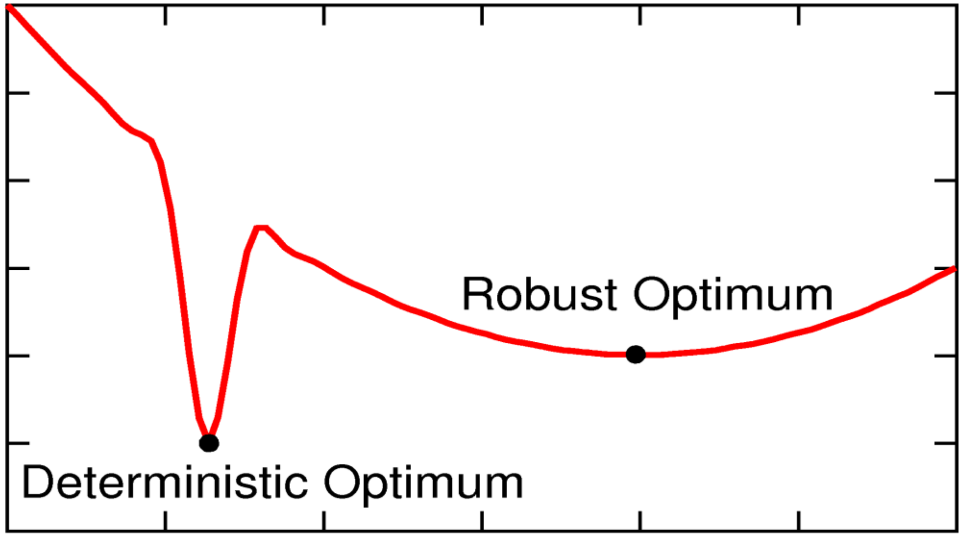
How to Define the Robustness of a Design?

- **Intuitively:** The performance of a robust design is largely unaffected by random perturbations
- **Variance indicator:** The coefficient of variation (CoV) of the objective function and/or constraint values is not greater than the CoV of the input variables
- **Sigma level:** Keep an undesired performance outside an interval of mean +/- sigma level (e.g. design for six-sigma)
- **Probability indicator (Reliability analysis):** The probability of reaching undesired performance is smaller than an acceptable value

⇨ Probability-based reliability analysis

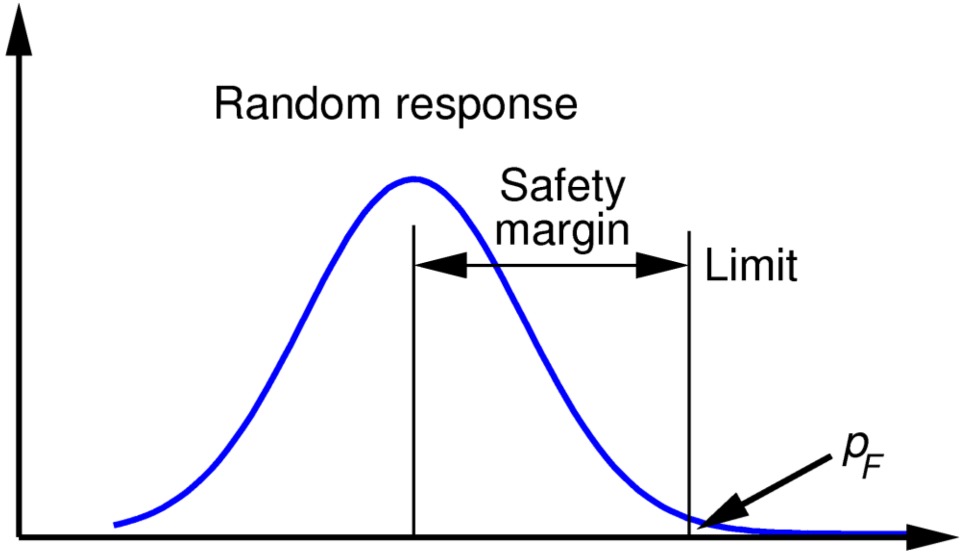
How to Define the Robustness of a Design?

Robustness in terms of stability



- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation)

Robustness in terms of requirements

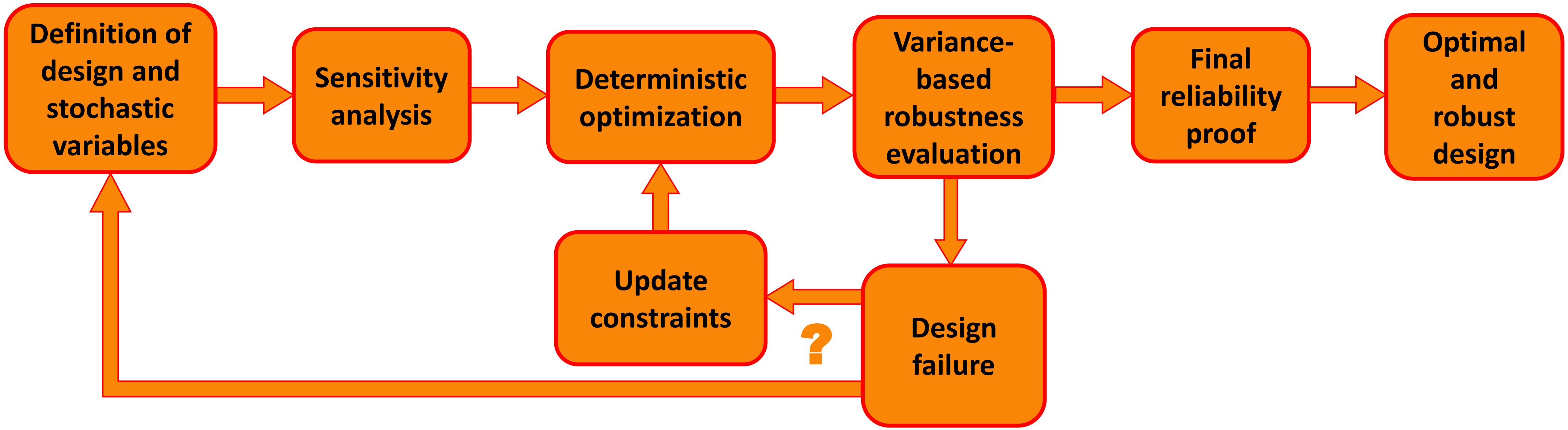


- Safety margin (sigma level) of one or more responses y:
- Reliability (failure probability) with respect to given limit state

Iterative (single objective) Robust Design Optimization

- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety factors are adjusted for the critical model responses

→ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design

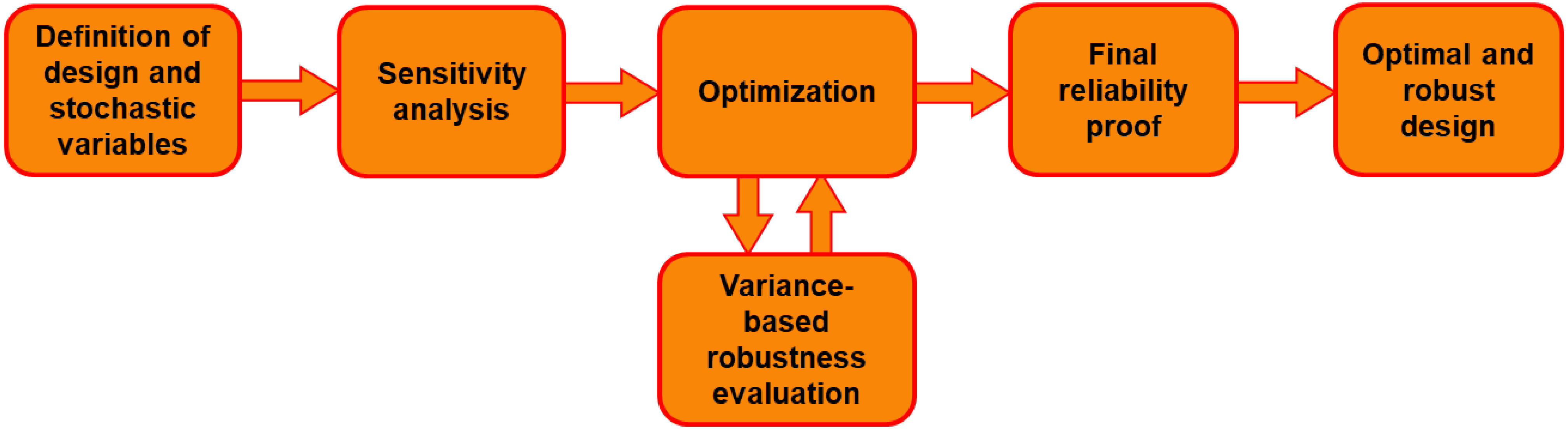


Coupled (single objective) Robust Design Optimization

- Fully coupled optimization and robustness/reliability analysis
- For each optimization (nominal) design the robustness/reliability analysis is performed

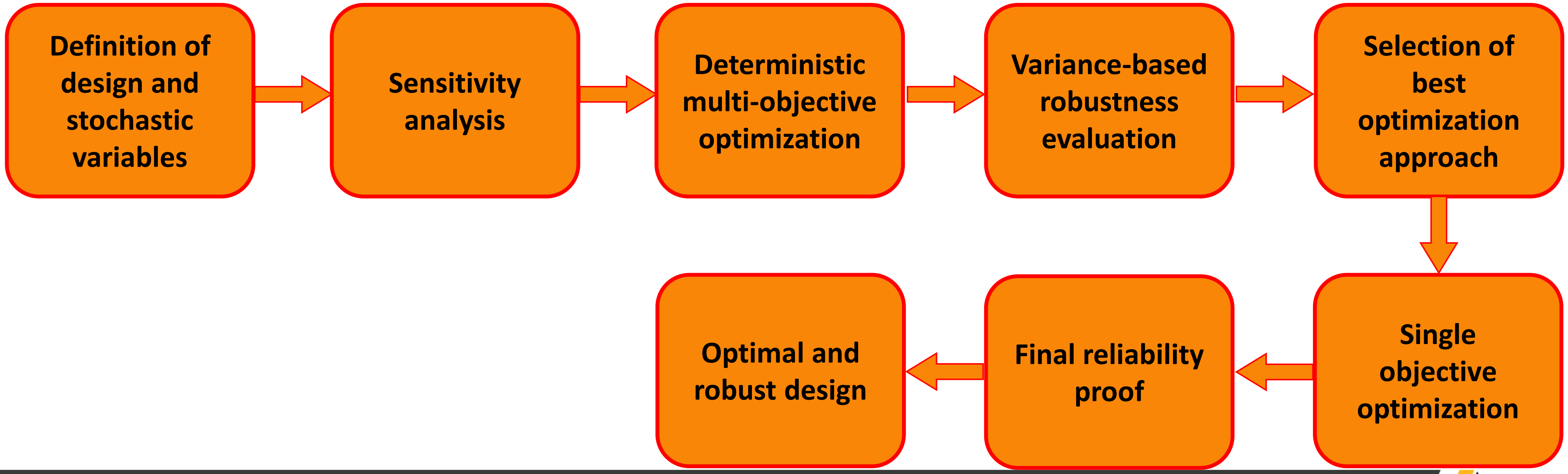
→ Implementation uses small sample variance-based robustness measures during the optimization (≥ 10 Designs) and a final (more accurate) reliability proof

→ But still the procedure is often not applicable to complex CAE models



Hybrid Robust Design Optimization

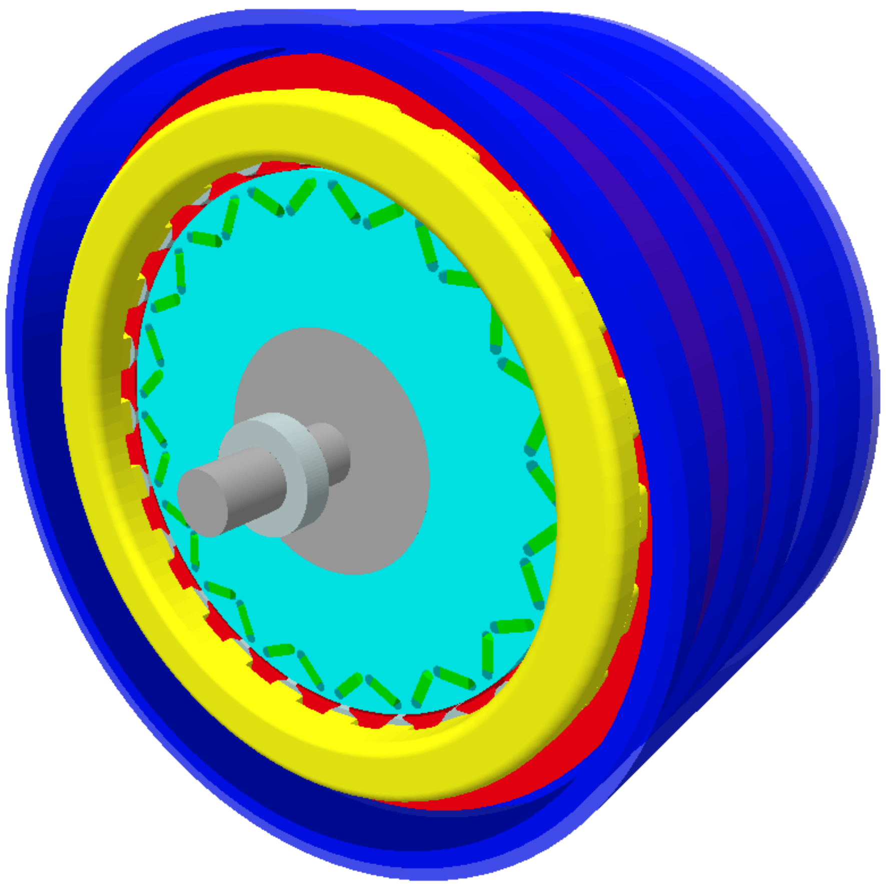
- Decoupled multi-objective optimization and robustness/reliability analysis
- For each validation design from the optimization run averaged performances are acquired
- Applicable to variance- and reliability-based RDO



Example

Specification

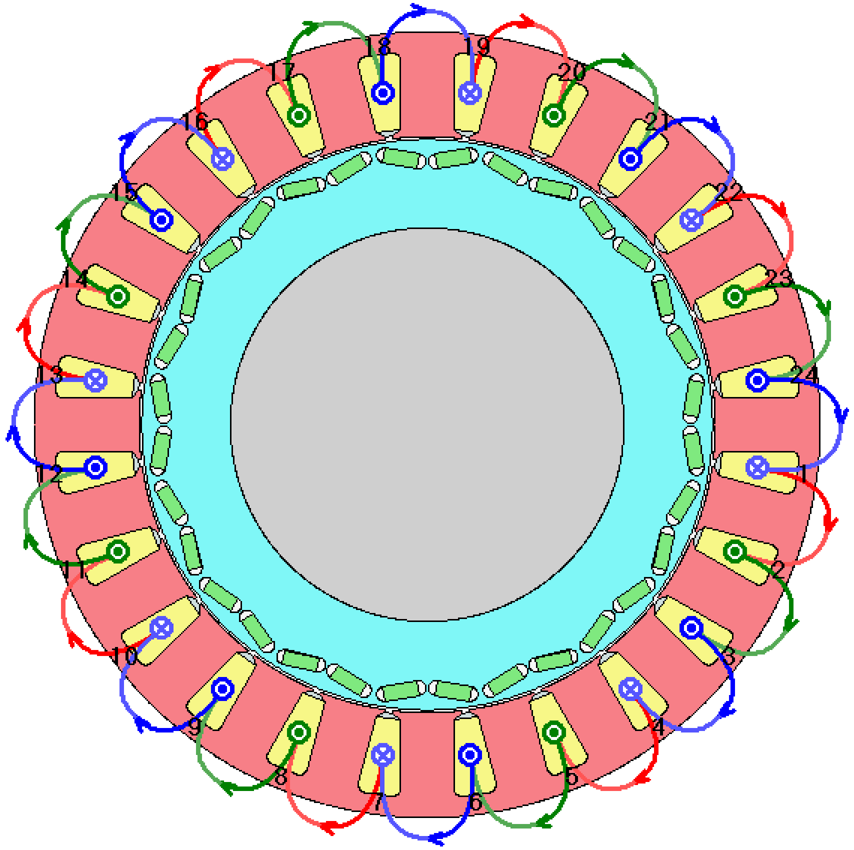
Requirement	Value	Unit
Peak torque	400	Nm
Peak power @ 3krpm, 6krpm	120, 100	kW
Cont. torque @ 1krpm, 5krpm	300, 124	Nm
Maximum speed	7000	Rpm
Cooling system	WJ	
Coolant flow rate	≤ 6.5	l/min
Coolant fluid type	EWG	
Coolant inlet temperature	65	°C
Line current	≤ 500	A_{rms}
DC bus voltage	350	V
Package envelope	330 (Φ) x 220	mm



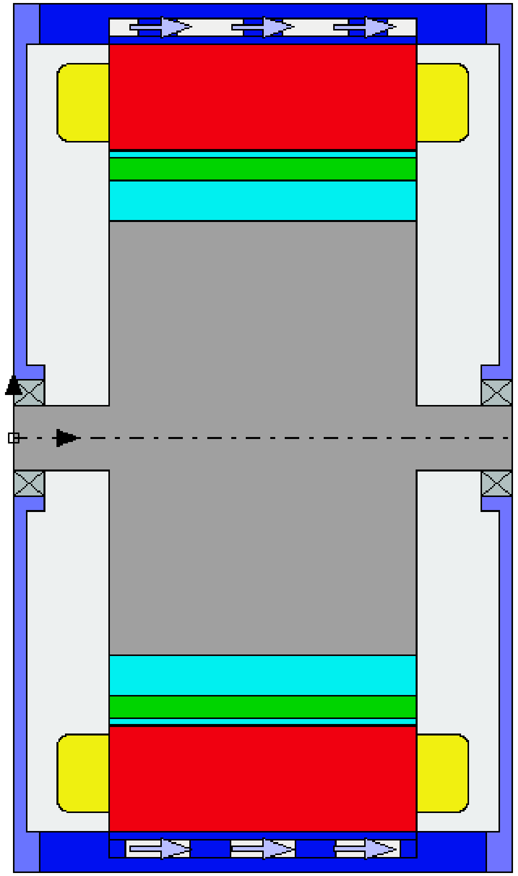
Concept Design

- **Machine topology:**
  - Stator slots = 24
  - Rotor poles = 16
  - V-shaped magnets
- **Materials:**
  - Magnets: N48UH
  - Magnetic cores: 235-35A
- **Winding:**
  - Double-layer, concentrated
  - Parallel paths per phase = 6
- **Geometry:**
  - Stator diameter (mm) = 300
  - Mechanical airgap (mm) = 1

Winding pattern



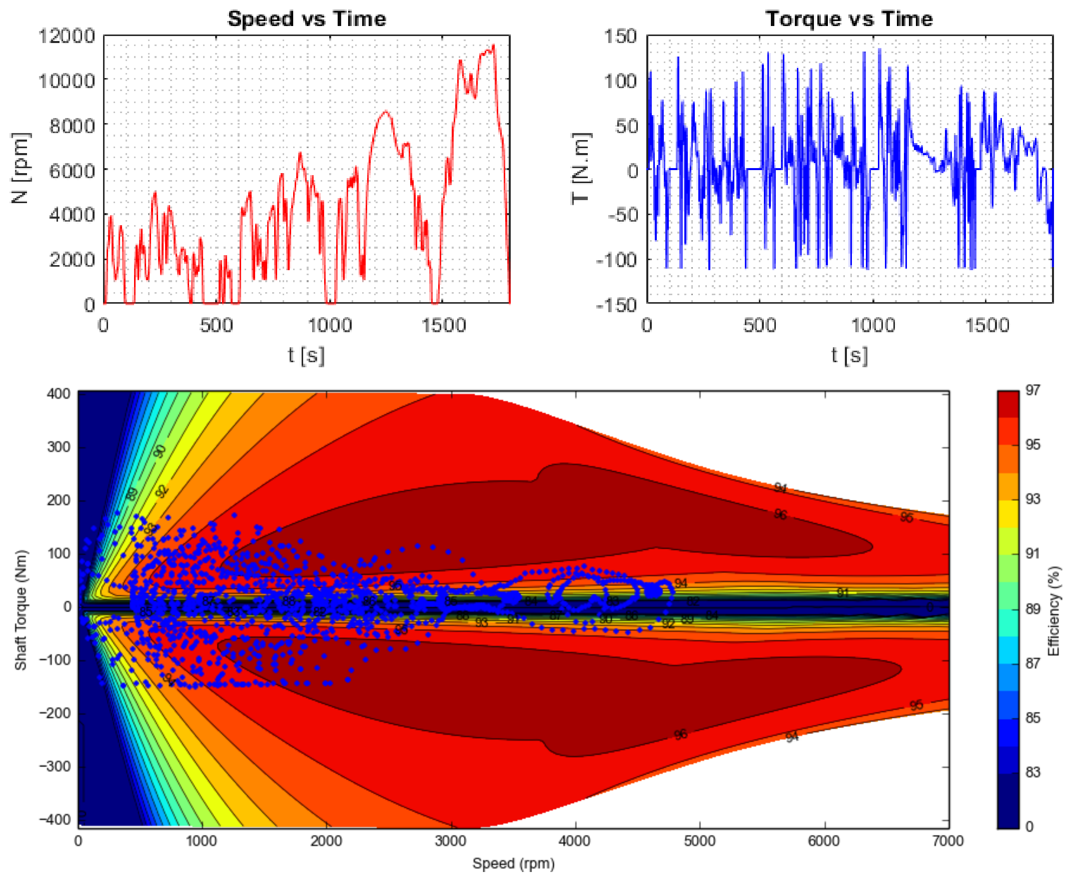
Cooling system



Optimization Scenario

- **Objectives:**
  - ❑ Maximum efficiency over WLTP-3
  - ❑ Minimum active volume
- **Constraints:**
  - ❑ Continuous torque (Nm) @ 1krpm ≥ 300
  - ❑ Continuous torque (Nm) @ 5krpm ≥ 124
  - ❑ Peak power (kW) @ 3krpm ≥ 120
  - ❑ Peak power (kW) @ 6krpm ≥ 100
  - ❑ Torque ripples (%) @ 1krpm ≤ 10
  - ❑ Von-Mises stress (MPa) @ 8.4krpm ≤ 300

WLTP-3 Drive Cycle



Initial Design Space

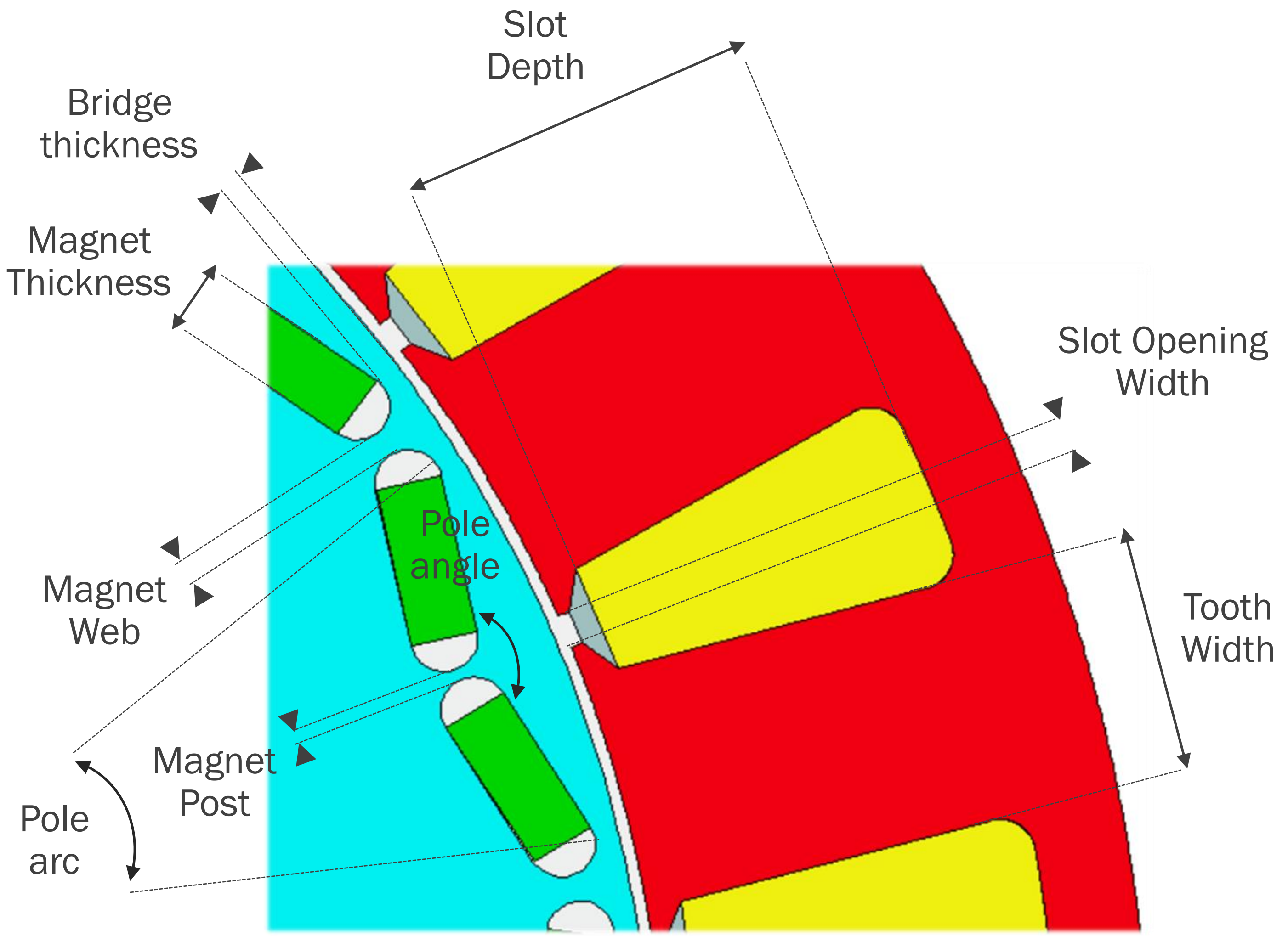
Parameter	Range	Unit
Active length	[70; 150]	mm
Bridge thickness	[0.4; 3]	mm
Magnet post	[0.4; 3]	mm
Magnet thickness	[4; 10]	mm
Pole arc ratio	[0.7; 1]	
Pole V angle	[90; 180]	°
Slot depth ratio[1]	[0.45; 0.8]	
Slot width ratio[2]	[0.4; 0.7]	
Split ratio[3]	[0.58; 0.85]	
Slot opening ratio[4]	[0.2; 0.8]	

[1] Slot Depth / (Slot Depth + Stator Back Iron Thickness)

[2] Slot Width / (Slot Width + Stator Tooth Width)

[3] Stator Inner Diameter/ Stator Outer Diameter

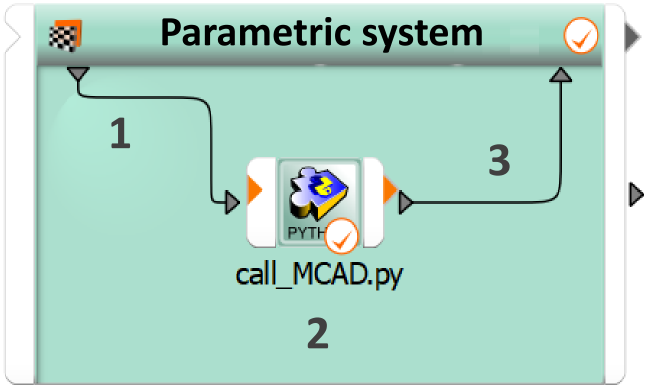
[4] Slot Opening Width/ Slot Pitch



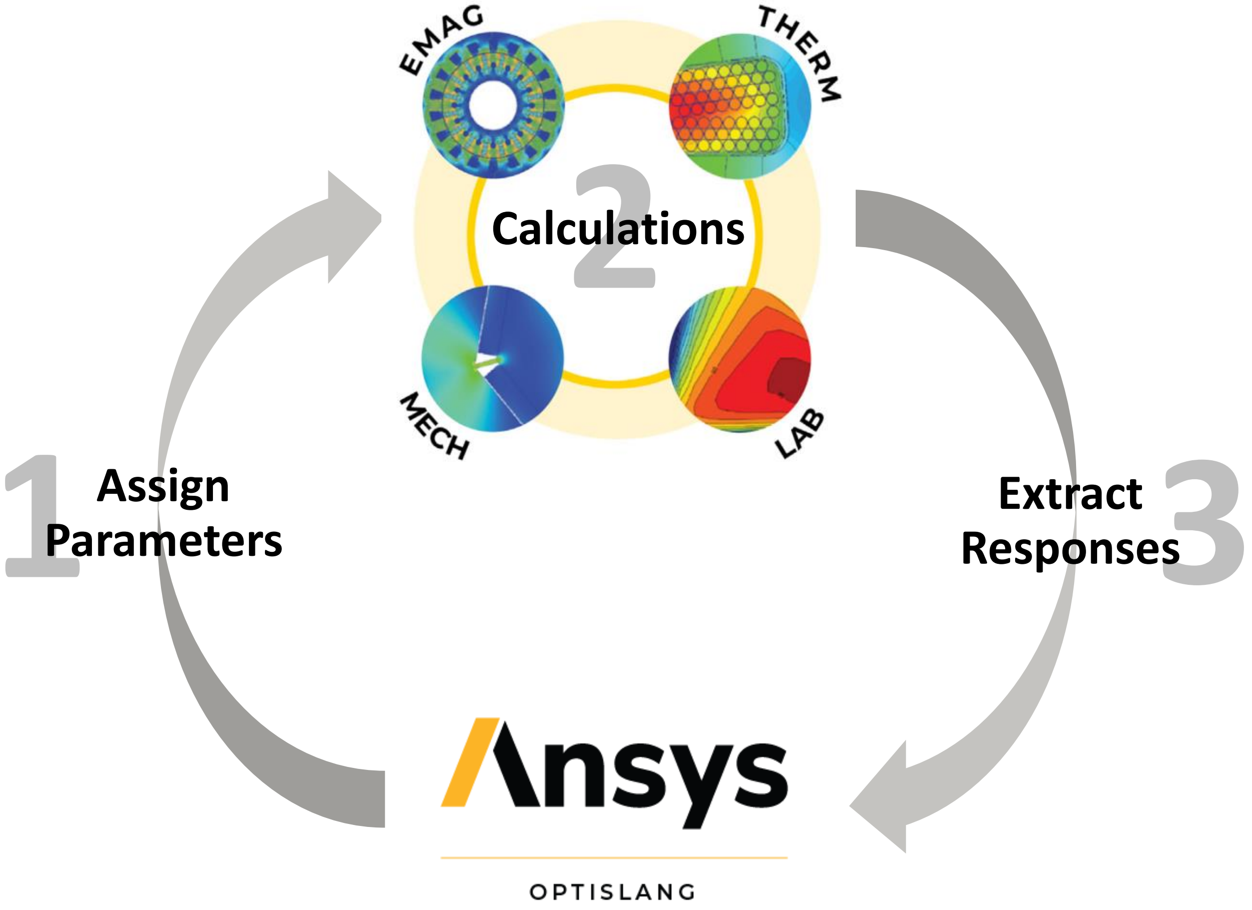
Ansys

Motor-CAD Integration

- **Motor-CAD is ActiveX driven through Python environment in order to:**
  1. Assign input design parameters
  2. Run multiphysics calculations
  3. Extract output performance data



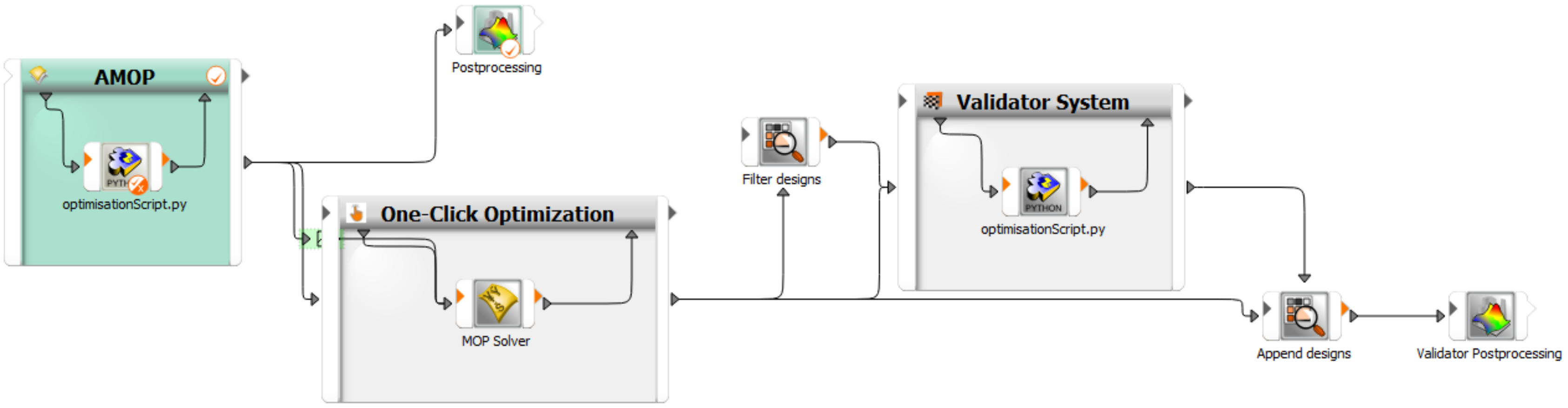
- **ActiveX controls can be found in Motor-CAD in one click.**



AMOP results

Multi-objective optimization

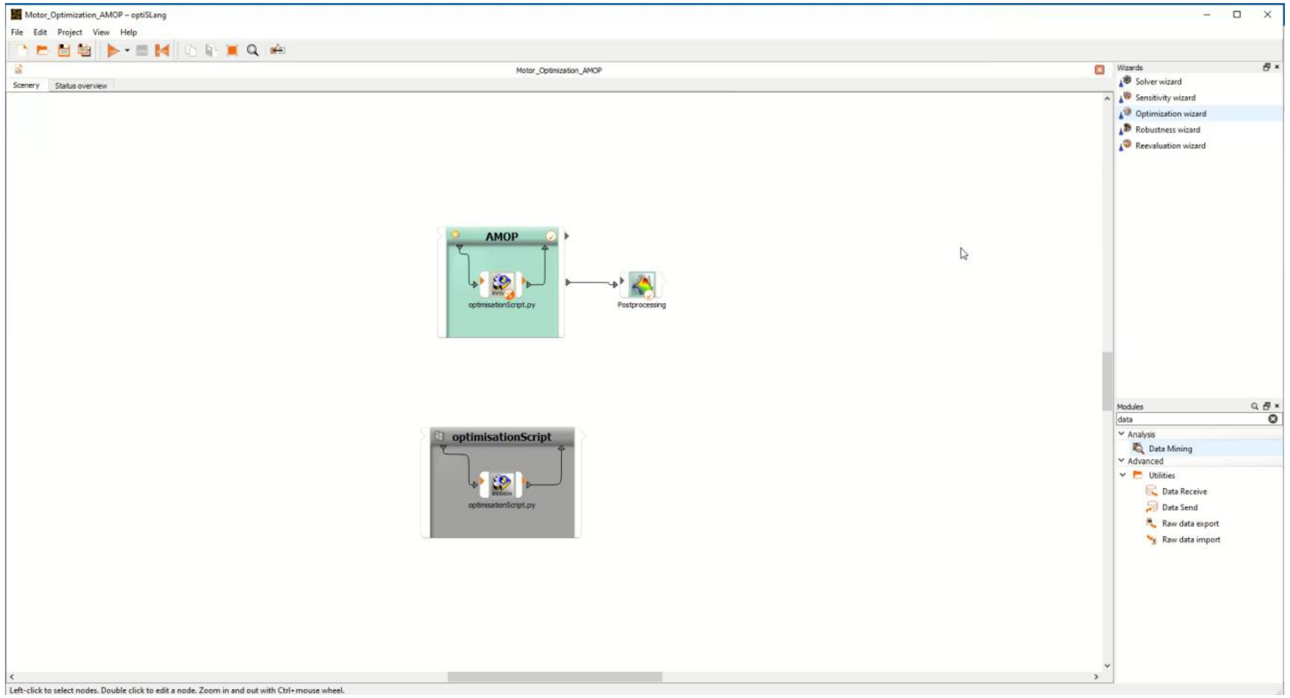
- Optimization Wizard to build Multi-objective optimization



Automatic Build Process with One-Click-Optimization and Validation

Variance-based robustness evaluation for best designs

- Optimization Wizard to build Multi-objective optimization
  - Add Robustness System to Validator System
  - “Last Chance” to add stochastic Properties for the Parameters



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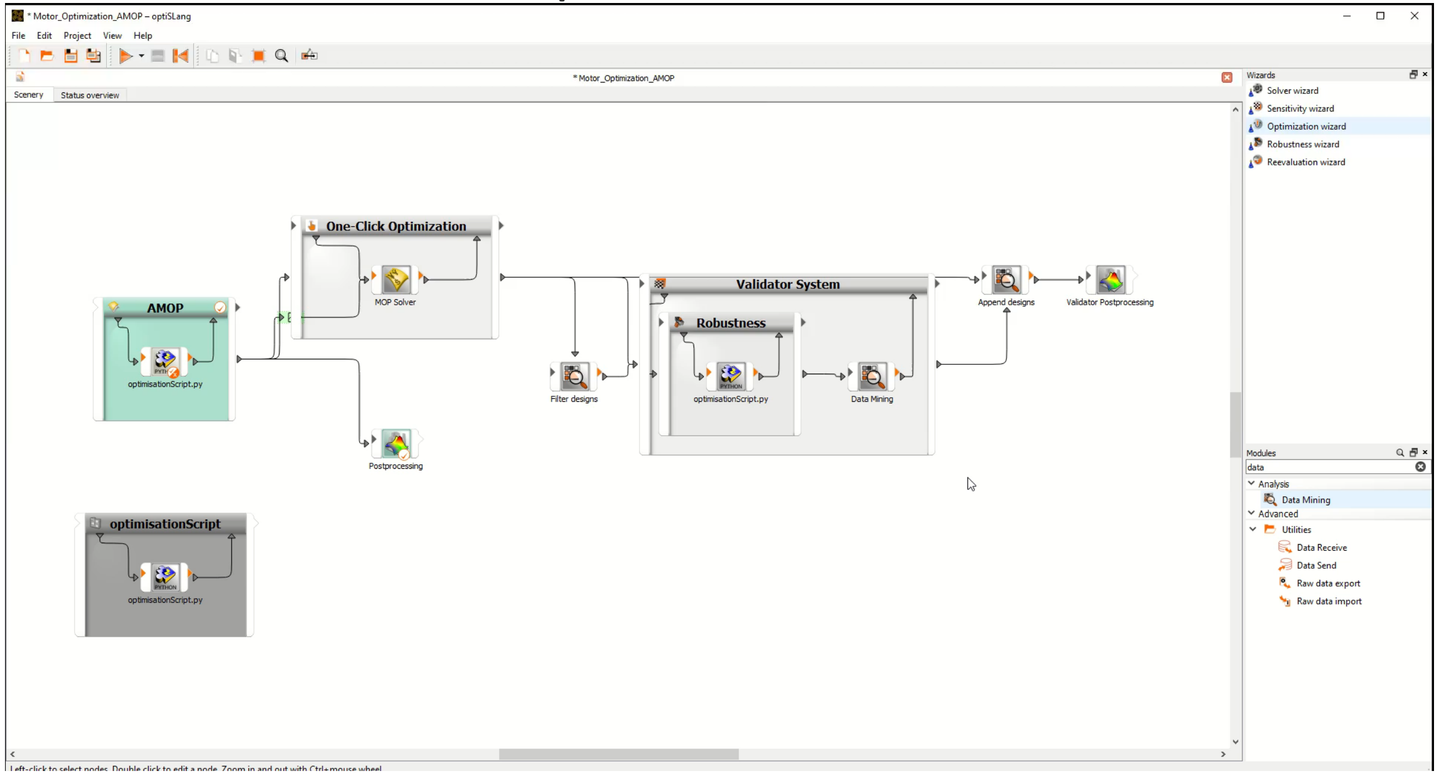
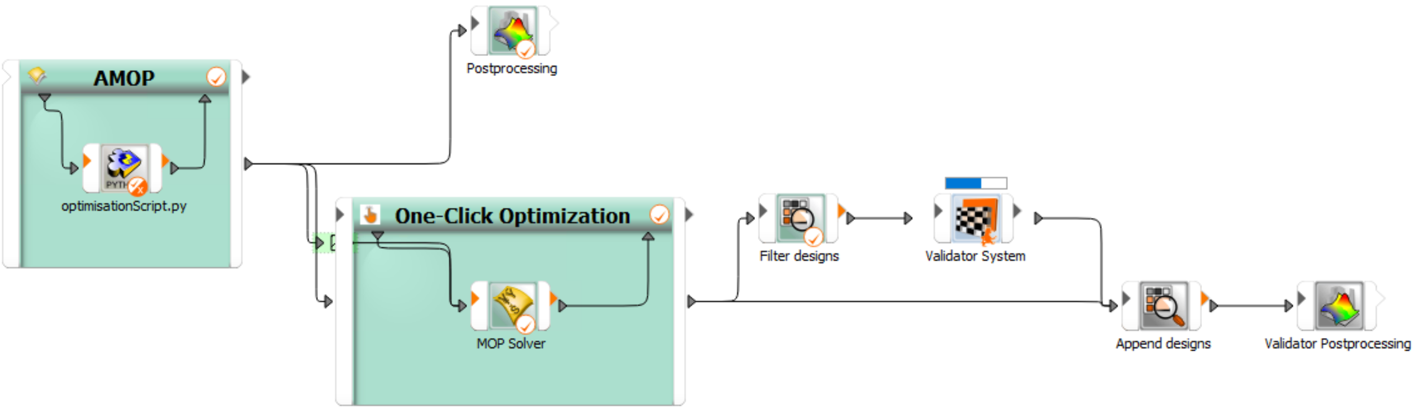


AMOP – with validated multi-objective optimization

- AMOP:
  - Refine MOP with objectives in mind
- One-Click-Optimization:
  - Multi-objective optimization
- Validator System
  - Validates few best design candidates



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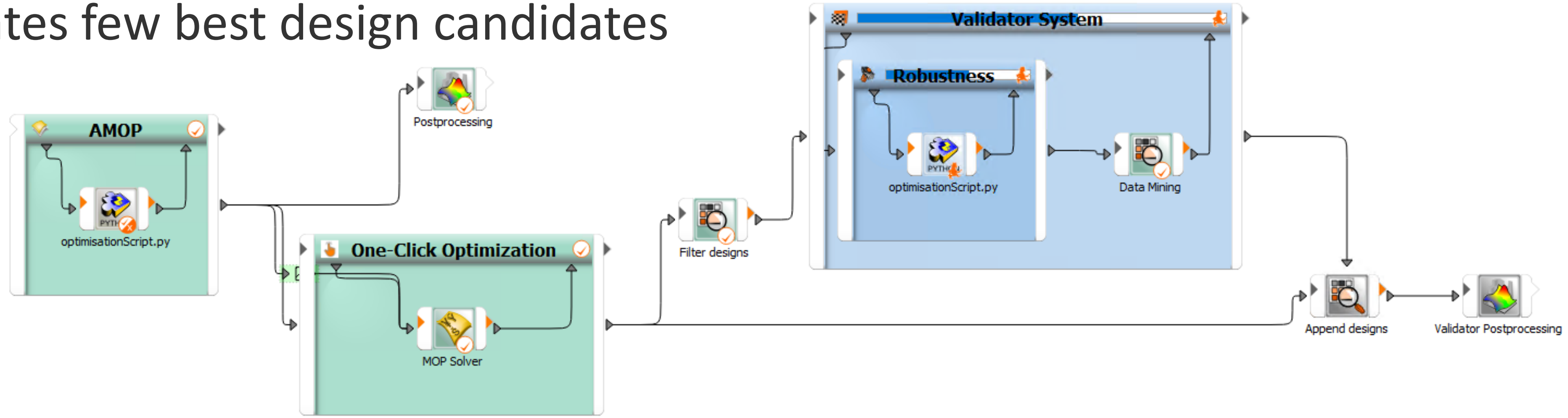

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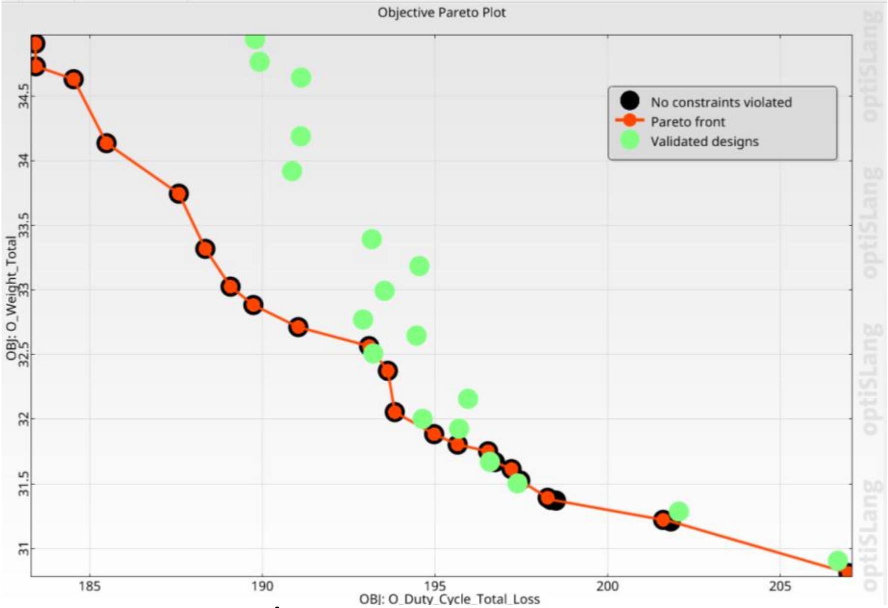
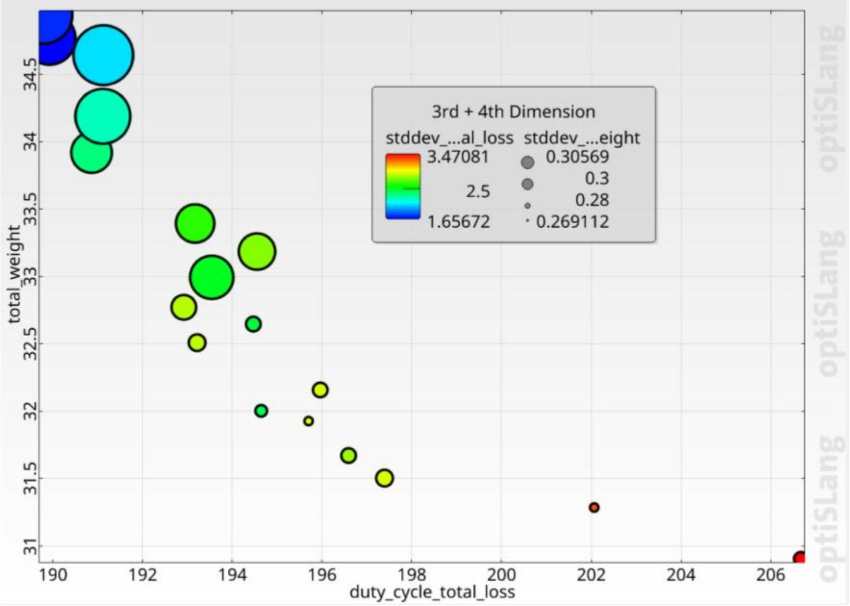
Validation results

Summary

- Hybrid Approach shows non-approximated Pareto front with Standard deviation
  - Compromise Evaluation can be done with Robustness in mind
  - Fragility Curves could be incorporated



Pareto-Front in Black/ Red as Approximation and Green from the Validation



Pareto-Front with Standard Deviation of the two objectives

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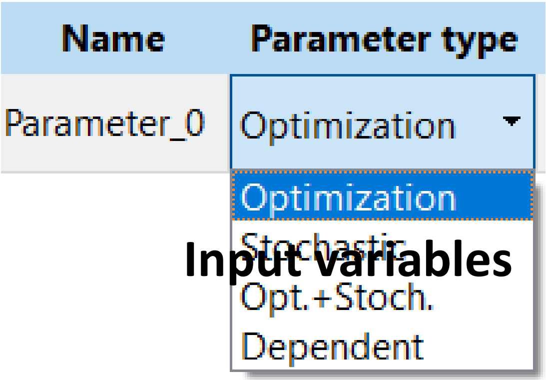
Process Integration

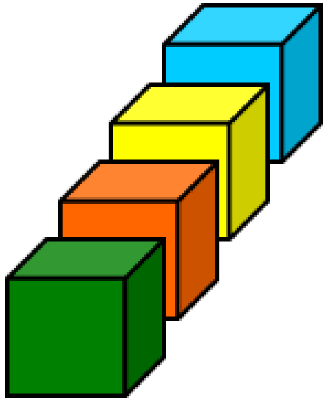
Parametric model as base for:

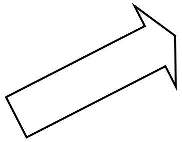
- User-defined optimization (design) space
- Naturally given robustness (random) space

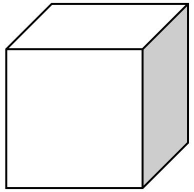
Design variables
Entities that define the design space
(Parameter type: Optimization



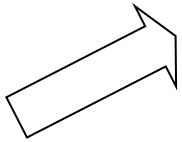
Input variables

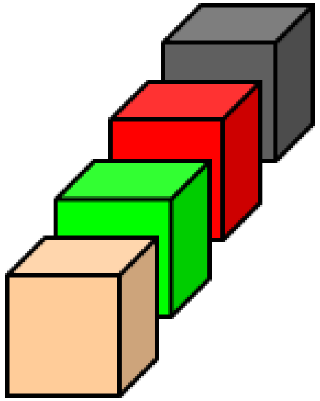






The CAE process generates the results according to the inputs





Response variables
Outputs from the system

Scattering variables
Entities that define the robustness space
(Parameter type: Stochastic)

Input and Response Variables

- **Scalar design variables**:
  - of value type REAL, INTEGER, STRING and BOOL
  - and with resolution of continuous, discrete and binary
- **Scalar stochastic variables** with continuous resolution

Value type	Resolution	Range		Range plot
REAL	Continuous	8	10	
STRING	Nominal discrete	steel; wood; aluminium		No order
INTEGER	Discrete by value	[0-100];		
BOOL	Ordinal discrete (by index)	false; true		

design variables

PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
	LOGNORMAL	0.02	0.01	50 %	-4.02359; 0.472381
	EXPONENTIAL	1	2	200 %	2; -1
	NORMAL	20	1	5 %	20; 1

stochastic variables

Input variables

Response variables
Numeric values only

- **Scalar responses** with continuous resolution
- **Vector responses** with continuous resolution having variable length
- **Signal responses** having variable length and several channels
- **2D and 3D field data** using enterprise add-on (Statistics on Structure SoS)

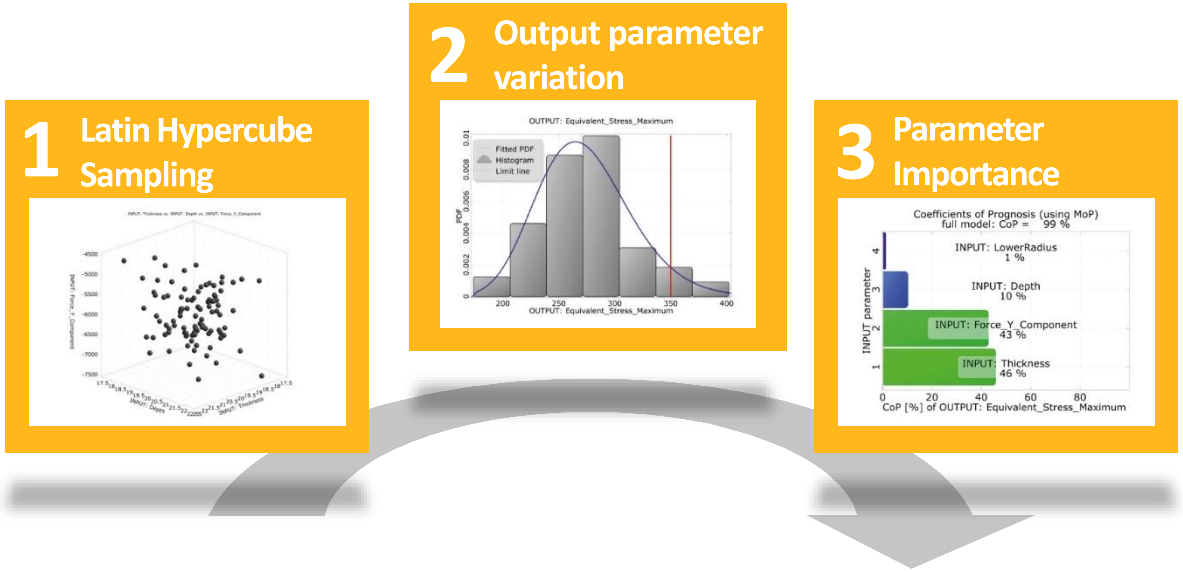
Verify Design Quality by Robustness Analysis

Before Scenario

- High entry barrier to start with robustness
- High simulation effort using standard Monte-Carlo approach
- Optimum not check against manufacturing scattering
- No quantification of risks
- No ranking of parameter scatter (Tolerances)

After Scenario

- Quantification of variable scatter as qualified input for subsequent stochastic analysis
- Easy setup of robustness & reliability analysis
- Automatic ranking of parameter scatter
- Less simulation even for small probability of failure



- **Powerful procedure to check design quality: Robustness evaluation with optimized DoE, Proof of Reliability with leading edge algorithms even for multiple failure regions.**
- **Statistical analysis of input correlations and fit of distribution functions.**
- **Guided wizards for easy and safe usage.**

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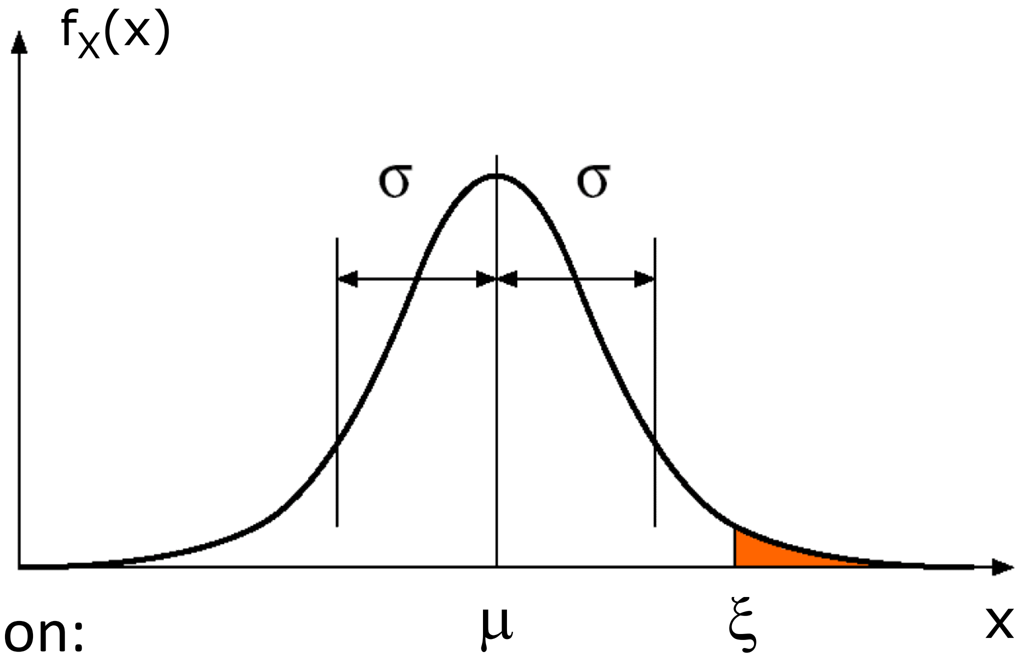
Exceedance Probability

- Probability of reaching values above a limit



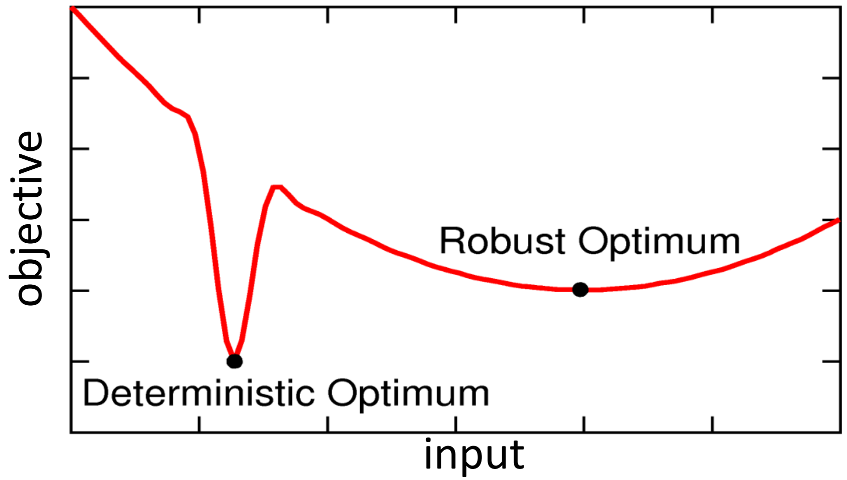
- For Gaussian distribution:

$$P_\xi = P[X \geq \xi]$$

ξ	μ	$\mu+\sigma$	$\mu+2\sigma$	$\mu+3\sigma$	$\mu+4\sigma$	$\mu+5\sigma$
P_ξ	$5.0 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-2}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-5}$	$2.9 \cdot 10^{-7}$

How to Define the Robustness of a Design?

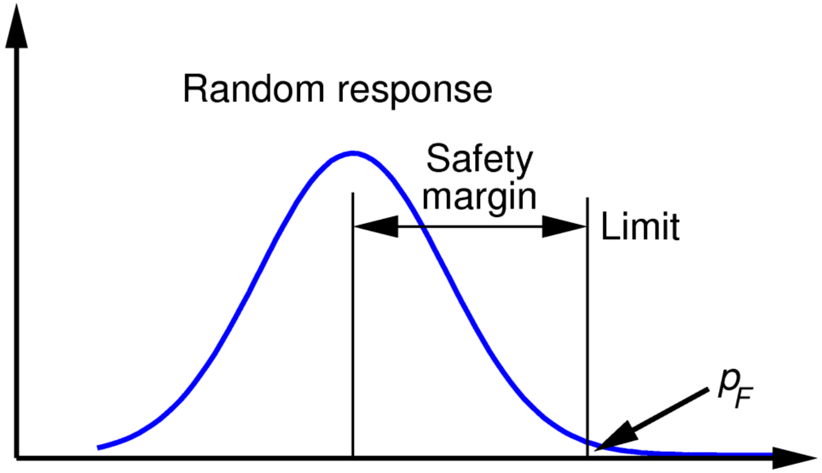
Robustness in terms of stability



- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Robustness in terms of requirements



- Safety margin (sigma level) of one or more responses y:

$$(y_{limit} - \mu_Y)/\sigma_Y \geq a$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

How to Define the Robustness of a Design?

- **Intuitively:** The performance of a robust design is largely unaffected by random perturbations
- **Variance indicator:** The coefficient of variation (CoV) of the objective function and/or constraint values is not greater than the CoV of the input variables
- **Sigma level:** Keep an undesired performance outside an interval of mean +/- sigma level (e.g. design for six-sigma)

Variance based robustness analysis

- **Probability indicator:** The probability of reaching undesired performance is smaller than an acceptable value

Probability based reliability analysis

How to quantify uncertainty?

- **Intuitively**: The performance of a robust design is largely unaffected by random perturbations
- **Variance indicator**: The coefficient of variation (CoV) of the objective function and/or constraint values is not greater than the CoV of the input variables
- **Sigma level**: Keep an undesired performance outside an interval of mean +/- sigma level (e.g. design for six-sigma)
- **Probability indicator**: The probability of reaching undesired performance is smaller than an acceptable value

Variance based robustness analysis

→ Probability based reliability analysis

increasing level of failure probability

not important

frequently

rarely ≈ 1/1000

very rarely

increasing level of uncertainty knowledge

unaware

estimated

qualified

increasing level of analysis complexity

low effort

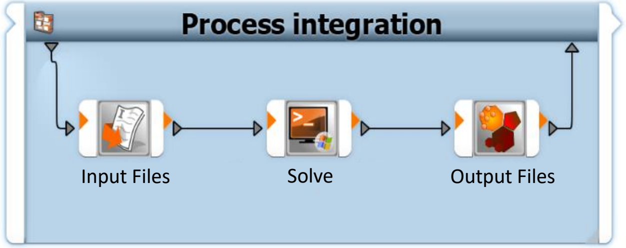
Independent limit(s) only

several limits / limit functions

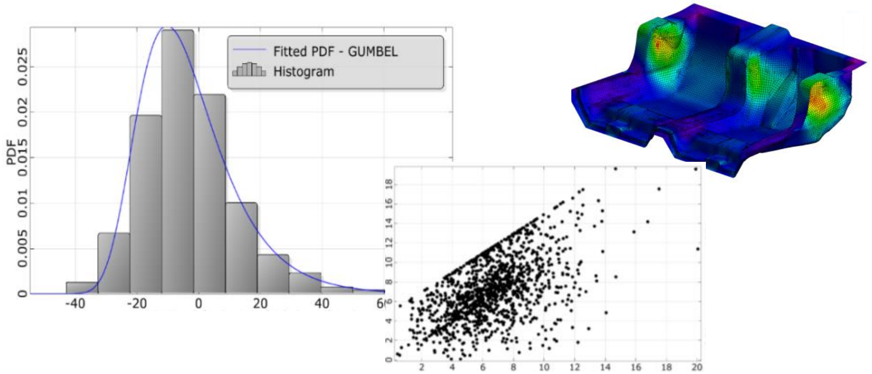
high effort

Variance and Reliability Based Robustness Analysis

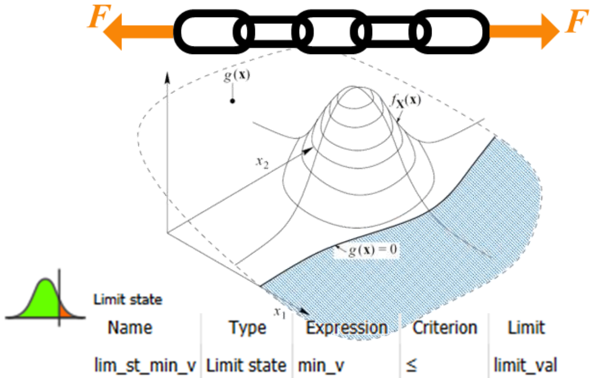
1. automate simulation workflow



2. derive and include parameter scatter and correlation

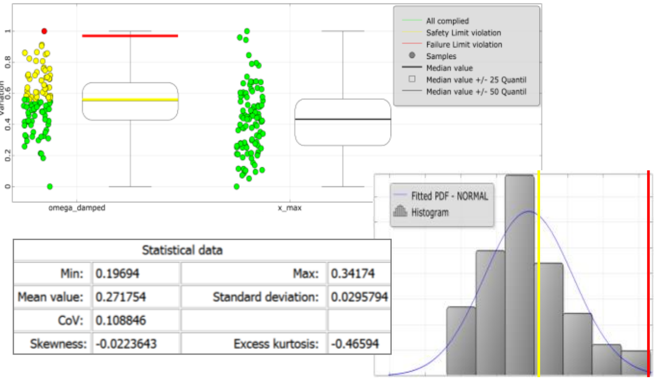


3.define limit state

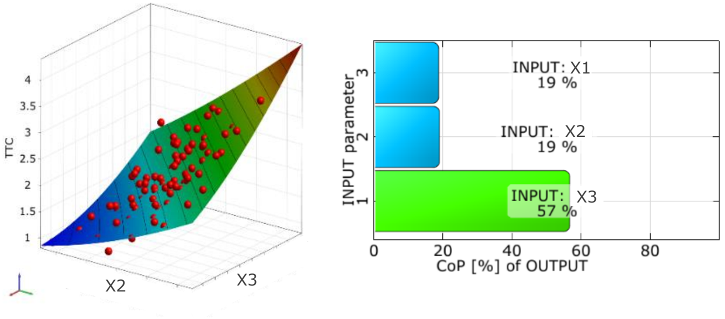


Run Analysis

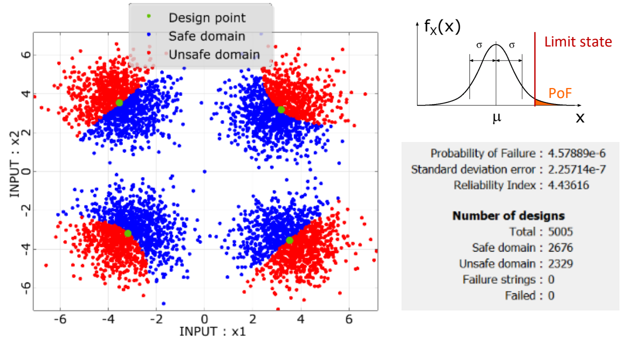
4. check the variation



5. parameter importance by robustness analysis



6. quantify uncertainty by reliability analysis



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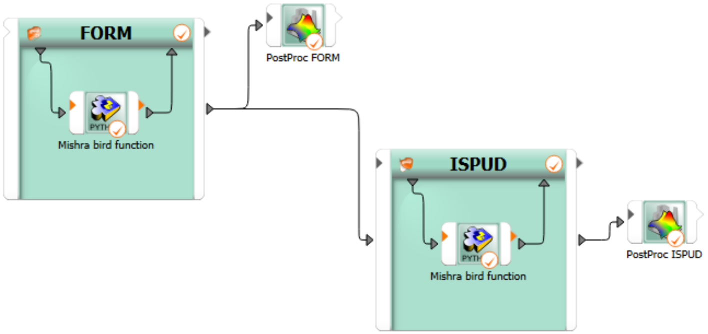
FORM (*) + ISPUD

(*) default settings



PoF = 1.3e-06

Method : First Order Reliability Method (FORM)

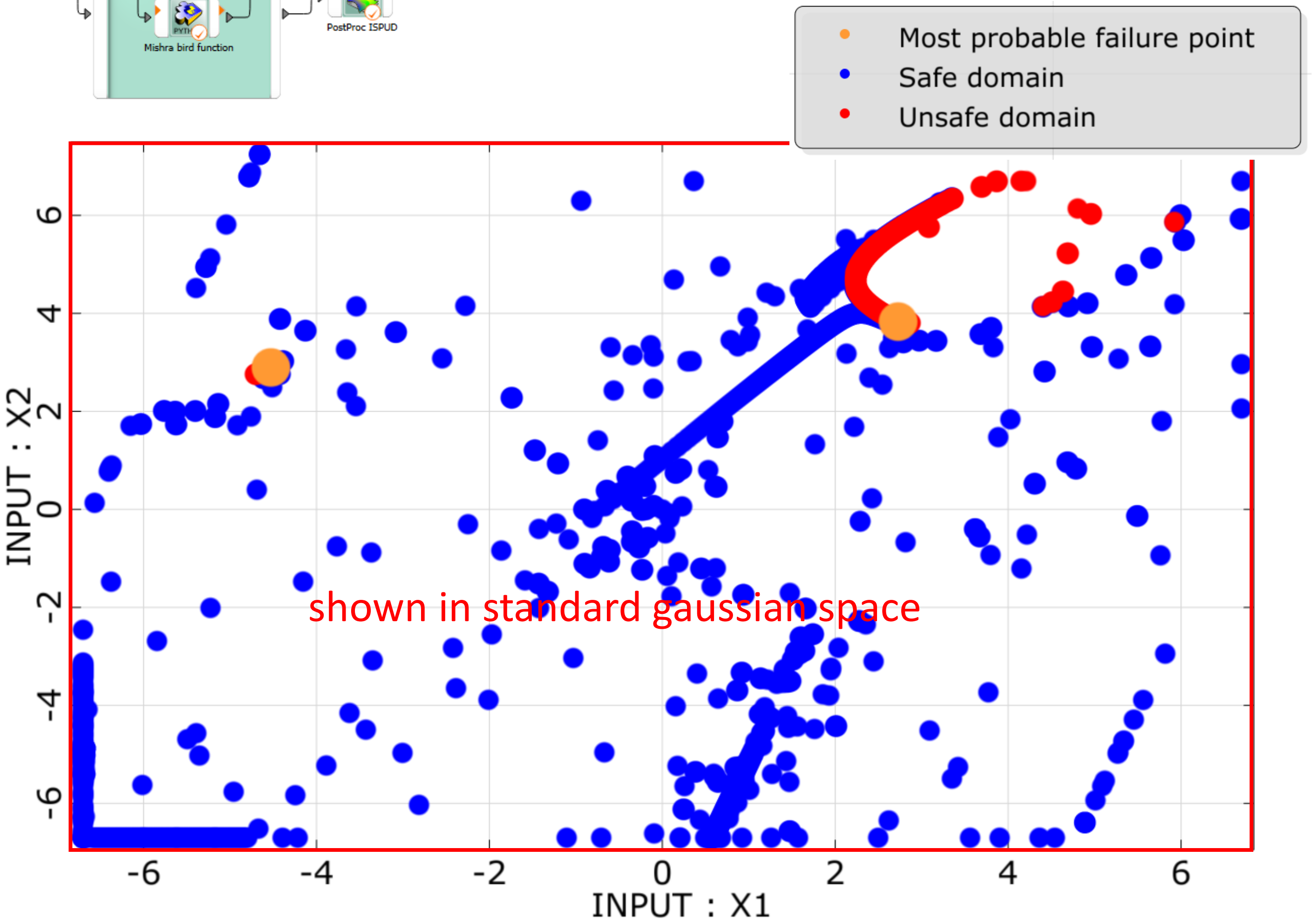
Probability of Failure : 1.3153e-06
Reliability Index : 4.69774

Number of designs

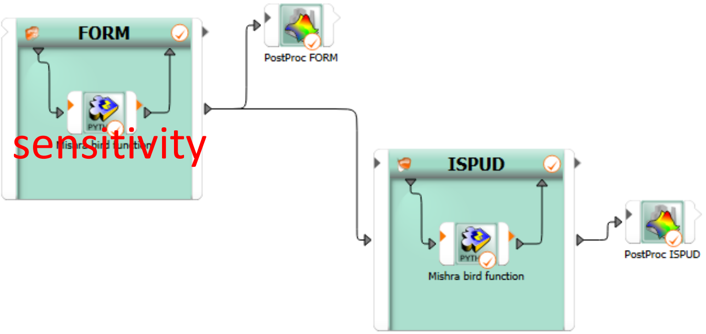
Total : 65881
Safe domain : 64634
Unsafe domain : 1247
Failure strings : 0
Failed : 0

Most probable failure point(s)

ID :	25415	20266
Input parameter values		
X1 :	-3.77261	-8.84384
X2 :	-2.60422	-1.79514
Reliability index (FORM) :	4.70378	5.37471
Probability of failure (FORM) :	1.27696e-06	3.83536e-08



FORM (*) + ISPUD



(*) 13 start design violating limit from sensitivity study, no further initial samples, desired accuracy = 0.005

PoF = 1.3e-06

Method : First Order Reliability Method (FORM)

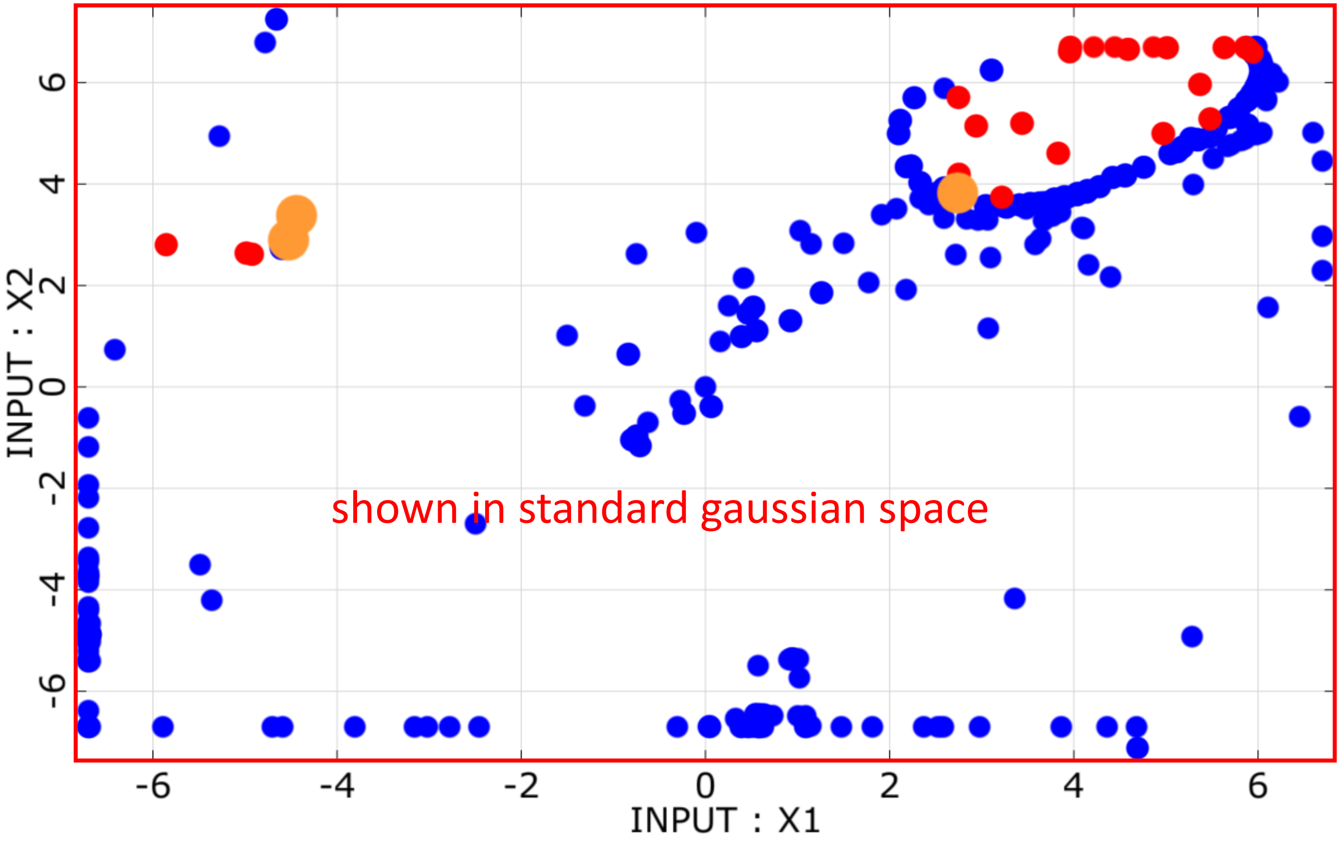
Probability of Failure : 1.31819e-06
Reliability Index : 4.69729

Number of designs

Total : 890
Safe domain : 734
Unsafe domain : 156
Failure strings : 0
Failed : 0

Most probable failure point(s)

ID :	406	886	455
Input parameter values			
X1 :	-3.76305	-8.84377	-8.87509
X2 :	-2.61315	-1.79501	-1.43022
Reliability index (FORM) :	4.70337	5.37468	5.58037
Probability of failure (FORM) :	1.2795e-06	3.83591e-08	1.20003e-08



Concrete Scenario 107 with most drastic TTC drop